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A NEGOTIATION PROCESS IMPLEMENTING THE NASH BARGAINING MODEL Part II

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In Suzuki (1989), a general payoff set was introduced, on the basis of Smale (1974a). This set is compact in R^2 but not necessarily convex. A bargaining model was then constructed, which is a generalized version of the noncooperative model in Nash (1953) and based on Smale (1974b). Although still a one-shot game, it was proved that under some regularity conditions, there are a finite number of extended Nash solutions. A generic property of the finiteness was also proved. That is, each element of an open and dense subset of the parameter set gives a finite number of extended Nash solutions. By using these results, the negotiation model in Rubinstein (1982) was studied.

The purpose of the present article is to examine the Rubinstein model in detail and to make a further discussion. Apart from the fact that this model is an infinite horizon game implementing the Nash model in a noncooperative way, it was shown that the subgame perfectness guarantees the uniqueness of the outcome for most cases discussed in the Rubinstein's article, where preferences of the players satisfy some specific conditions. These results were applied to an asymmetric bargaining model in Shaked and Sutton (1984), where a simple existence theorem for the Rubinstein model was also supplied. Another extension was made by Binmore, where a random move and a "not steadily shrinking cake" case are involved. The discussion is contained in Chapter 5 of Binmore and Dasgupta (1987).

It should be noted, however, that these models can have some problems if more than two players are negotiating, or information is not complete, as summarized in Chapter 7 of van Damme (1987). Concerning the Independence of Irrelevant Alternatives, it is obvious that the Rubinstein model (and its immediate derivatives) can not be compatible with this controversial axiom, because the solution depends on the process and which player moves first. However, as noted at the end of the Introduction of Binmore and Dasgupta (1987), the discussion could survive in a modified form.

1. The Rubinstein Model

Two players, 1 and 2, are bargaining on the partition of a pie, or formally $S=[0, 1]$. Each player makes an offer alternately, and the offer can be accepted (y) or rejected (n) by the partner. This process continues until an agreement, if any, is attained. No action is subject to any previous action. It is also assumed that no uncertainty is involved.

The set F of all strategies of the player who starts the bargaining is defined in the following way:

$f=\{f^t\}$, $t \in N$, belongs to F
if and only if
$f^1 \in S$,
$f^t: S^{t-1} \longrightarrow S$, for t odd, and
$f^t: S^t \longrightarrow \{y, n\}$, for t even.

N is the set of natural numbers, and S^t is the Cartesian product of S with itself t times. Similarly, the set G of all strategies of the player who is to respond to the first move of the partner is defined:

$g=\{g^t\}$, $t \in N$, belongs to G
if and only if
$g^t: S^t \longrightarrow \{y, n\}$, for t odd, and

g^t: $S^{t-1} \longrightarrow S$, for t even.

F×G can be considered as the set of strategy pairs, where player 1 starts the bargaining. Take (f, g)∈F×G, and suppose that an agreement is reached at t=T(f, g), and player 1 receives S=D(f, g) ∈S. Player 2 receives 1−s by definition. D(f, g) is called the partition induced by (f, g). The outcome function P is defined by:

$P: F\times G \longrightarrow S\times N \cup \{(0, \infty)\}$
such that
$P(f, g)=(D(f, g), T(f, g))$, if $T(f, g)<\infty$,
otherwise
$P(f, g)=(0, \infty)$

$(0, \infty)$ means a perpetual disagreement. The case where player 2 starts the bargaining can be defined in a similar manner. It should be noted that by smoothing the maps f^t and g^t, t∈N, and imposing the Whitney topology into F and G, as discussed in Suzuki (1989), a generic stability of outcomes could be obtained. This will be studied in another article.

Concerning the preference of player i, i=1, 2, Rubinstein makes the following assumptions.

Assumption 1

Player i has a complete, reflexive, and transitive preference relation on $S\times N \cup \{(0, \infty)\}$ which also satisfies:

For all r, s∈S, t, t_1, t_2∈N,

a) if $r_i > s_i$, then (r, t) Pi (s, t);
b) if $s_i > 0$ and $t_2 > t_1$, then
   (s, t_1) Pi (s, t_2) Pi $(0, \infty)$;
c) (r, t_1) Ri (s, t_1+1)
   if and only if

(r, t_2) Ri (s, t_2+1);

d) given $s \in S$, t_1 $t_2 \in N$,
the sets $\{r \in S \mid (r, t_1) \text{ Ri } (s, t_2)\}$
and $\{r \in S \mid (r, t_1) \text{ Ri } (0, \infty)\}$
are closed in S;

e) if $(s+\alpha, 1)$ Ii $(s, 0)$,
$(s'+\alpha', 1)$ Ii $(s', 0)$, and $s_1 < s'_1$,
then $\alpha_1 \leq \alpha'_1$.

Concerning the notation, r_1 is the portion of S for player 1, i.e. $r_1=r$, and $r_2=1-r$, etc. aRib means that a is at least as good as b for player i. aPib means aRib but not bRia. aIib means aRib and bRia. It will be seen that e) is not indispensable for a general discussion. From c), the expression (r, T) Ri $(s, 0)$ can be used for $(r, T+t)$ Ri (s, t).

Two models are then presented by Rubinstein, where the preferences satisfy these assumptions:

1) Fixed bargaining costs
Player i, $i=1, 2$, has a number c_i such that
(r, t_1) Ri (s, t_2) if and only if
$(r_i - c_i t_1) \geq (s_i - c_i t_2)$.

2) Fixed discounting factors
Player i, $i=1, 2$, has a number $0 < \delta_i \leq 1$ such that
(r, t_1) Ri (s, t_2) if and only if
$r_i \delta^{t_1} \geq s_i \delta^{t_2}$.

Definition 1

$(f^*, g^*) \in F \times G$ is called a Nash Equilibrium if there is no $f \in F$ such that $P(f, g^*) P_1 P(f^*, g^*)$, and no $g \in G$ such that $P(f^*, g) P_2 P(f^*, g^*)$.

As shown by Rubinstein, this definition is too weak, because every

$s \in S$ can be an equilibrium partition. As a stronger concept, Rubinstein adopts the subgame perfectness. A detailed discussion of this concept can be found in Chapters 6 and 8 of van Damme (1987).

Take any vector $(s^1, ..., s^T) \in S^T$ and $f \in F$. $f \mid s^1 ... s^T$ is defined to be a strategy derived from f after the history of offers $(s^1, ..., s^T)$.

For example, if T and t are odd:

$$(f \mid s^1 ... s^T)^t (r^1 ... r^t)$$
$$= f^{T+t}(s^1 ... s^T, r^1 ... r^t)$$

It should be noted that if T is odd, $f \mid s^1 ... s^T \in G$. $g \mid s^1 ... s^T$ can be defined in a similar manner.

Definition 2

$(f^*, g^*) \in F \times G$ is called a Perfect Equilibrium (PE) if for all $(s^1 ... s^T)$, T odd:

1) there is no $f \in F$ such that
$P(f^* \mid s^1 ... s^T, f) P_2 P(f^* \mid s^1 ... s^T, g^* \mid s^1 ... s^T)$;
2) if $g^{*T}(s^1 ... s^T) = y$, there is no $f \in F$ such that
$P(f^* \mid s^1 ... s^T, f) P_2 (s^T, 0)$;
3) if $g^{*T}(s^1 ... s^T) = n$, then
$P(f^* \mid s^1 ... s^T, g^* \mid s^1 ... s^T) R_2 (s^T, 0)$;

and if T is even:

4) there is no $f \in F$ such that
$P(f, g^* \mid s^1 ... s^T) P_1 P(f^* \mid s^1 ... s^T, g^* \mid s^1 ... s^T)$;
5) if $f^{*T}(s^1 ... s^T) = y$, there is no $f \in F$ such that
$P(f, g^* \mid s^1 ... s^T) P_1 P(s^T, 0)$;
6) if $f^{*T}(s^1 ... s^T) = n$, then
$P(f^* \mid s^1 ... s^T, g^* \mid s^1 ... s^T) R_1 (s^T, 0)$

Define:

$A=\{s \in S \mid$ there is a PE $(f, g) \in F \times G$ such that $s=D(f, g)\}$

$B=\{s \in S \mid$ there is a PE $(g, f) \in G \times F$ such that $s=D(g, f)\}$

On the basis of these settings, the following lemmas are proved.

Lemma 1

Let $a \in A$. For all $b \in S$ such that $b>a$, there is $c \in B$ such that $(c, 1) R_2 (b, 0)$.

Lemma 2

For all $a \in B$ and all $b \in S$ such that $b<a$, there is $c \in A$ such that $(c, 1) R_1 (b, 0)$.

Lemma 3

Let $a \in A$. Then for all $b \in S$ such that $(b, 1) P_2 (a, 0)$, there is $c \in A$ such that $(c, 1) R_1 (b, 0)$.

Lemma 4

Let $a \in B$. Then for all $b \in S$ such that $(b, 1) P_1 (a, 0)$, there is $c \in A$ such that $(c, 1) R_2 (b, 0)$.

Define:

$\triangle=\{(x, y) \in S \times S \mid$ y is the smallest number such that $(y, 0) R_1 (x, 1)$, and x is the largest number such that $(x, 0) R_2 (y, 1)\}$

$\triangle_1=\pi_1(\triangle)$, where π_1: $S \times S \longrightarrow S$ such that π_1: $(x, y) \mid\longrightarrow x$;

$\triangle_2=\pi_2(\triangle)$, where π_2: $S \times S \longrightarrow S$ such that π_2: $(x, y) \mid\longrightarrow y$.

Then it is proved that if $(x, y) \in \triangle$, then $x \in A$ and $y \in B$ (Proposition 1), and that $\triangle$ is nonempty (Proposition 2). However, in order to reach the conclusion that $A = \triangle_1$, and $B = \triangle_2$, a further discussion must be made.

2. The Structure of $\triangle$

By definition, $\triangle$ is the whole collection of $(x, y) \in S \times S$ such that $y = d_1(x)$ and $d_2(d_1(x)) = x$, where:

$d_1: S \longrightarrow S$
such that
$d_1(x) = \text{Min } \{y \in S \mid (y, 0) R_1 (x, 1)\}$
and
$d_2: S \longrightarrow S$
such that
$d_2(y) = \text{Max } \{x \in S \mid (x, 0) R_2 (y, 1)\}$

From assumption 1 d), d_1 and d_2 are well defined and continuous. Assumption 1 a) gives that the maps are increasing, and atrictly increasing where $d_1(x) > 0$ and $d_2(y) < 1$. The continuity makes $\triangle$ closed in $S \times S$. Recalling that S is compact, $\triangle$ is in fact a compact subset of $S \times S$. However, $\triangle$ is not necessarily connected, unless, for example, d_1 and d_2 are linear on $\triangle_1$ and $\triangle_2$ respectively. Assumption 1 e) is used to show that every point of $\triangle$ has the same distance from the diagonal of $S \times S$. So without this assumption, Proposition 3 by Rubinstein can only state that $\triangle$ is closed. The proof of Proposition 4 should also be corrected, because it is based on the connectedness of $\triangle$.

Proposition 4

If $a \in A$, then $a \in \triangle_1$, and if $b \in B$, then $b \in \triangle_2$.

Proof

Consider first $\triangle_1$ and A. From Proposition 1, $\triangle_1$ is a subset of A. Since $\triangle_1 = \pi_1(\triangle)$, $\triangle_1$ is compact, so that $x_1 = \mathrm{Min}\ \triangle_1$ and $x_2 = \mathrm{Max}\ \triangle_1$ exist. Lemma 1 gives that $x_1 = \mathrm{Inf}\ A$, and Lemma 3 gives that $x_2 = \mathrm{Sup}\ A$, as shown by Rubinstein. If $\triangle$ is not connected, a further discussion is needed.

Suppose that there is $x^* \in A$ which does not belong to $\triangle_1$. Then $x_1 < x^* < x_2$, and there is a neighborhood Nx^* in S such that the intersection of Nx^* and $\triangle_1$ is empty. By definition, there is $(f^*, g^*) \in F \times G$ such that $D(f^*, g^*) = x^*$, and it is possible to find (f^*, g^*) which also satisfies $D(f^*\ b, g^*\ b) = d_2^{-1}(b)$ for all $b \in S$, $b > x^*$. Then if $d_2(d_1(x^*)) < x^*$, the Rubinstein's proof of Proposition 4 also holds locally, by using $s' = \mathrm{Sup}\ \{A \cap [0, x^* + \alpha]\}$, where $\alpha > 0$ is chosen to make $x^* + \alpha \in Nx^*$.

If $d_2(d_1(x^*)) > x^*$, a similar conclusion can be derived by using $t' = \mathrm{Inf}\{A \cap [x^* - \alpha]\}$.

Therefore, it can be concluded that there is no $x^* \in A$ which does not belong to $\triangle_1$. Similarly, there is no $y^* \in B$ which does not belong to $\triangle_2$.

This completes the proof.

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ナッシュ交渉モデルの補完に関して：第2部

〈要　約〉

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本稿においてはSuzuki (1989) に引き続いてRubinstein (1982) で展開されている交渉の数理モデルが考察の対象となっている。

第1節ではこの Rubinstein model が詳しく解説されており，特に subgame perfectness の概念が単なる Nash Equilibrium を補完する重要な役割を担っていることが示される。

第2節では上記 Rubinstein の論文中の Proposition 4 の証明が問題を含むことが指摘され，△が connected でない場合でもおなじ結論が導かれるとの追加証明がなされる。

プレーヤーの戦略の集合，FとG，を C^r 関数空間とし，C^r Whitney topology を入れて構造安定性などを導く試みは稿を改めて行う予定である。