GENERALIZED URBAN LAND RENT FUNCTION
– Empirical Investigation : A Case of Tokyo –

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I Introduction

Spatial distribution of urban land rent is explained as a negative exponential function of the distance from the central business district (CBD) in equilibrium models of urban rent such as those in Muth, Mills and others.[1] Negative exponential urban land rent function (NEULRF) is common to urban economists because it is easy to estimate. However, NEULRF is one of the special cases of more general but complicated functional form which is derived from the same equilibrium model.

Recently Kau and Sirmans applied the Box-Cox transformation technique to the functional form of urban rent.[2] They tested the negative exponential and the generalized functional relationship by using Chicago historical data. Their conclusion was that NEULRF proved to be the correct form in 2 of the 6 years tested.

The main purpose of this paper is to examine the functional form of urban land rent in Tokyo and to discuss the price elasticity of demand for housing and the elasticity of rent with respect to distance. As for the estimation procedure we use the maximum likelihood method to determine the transformation parameter as Kau and Sirmans did. However, we introduced another concept to evaluate the fitting of the estimated equation.

II The Negative Exponential Urban Land Rent Function

In this section, we derive NEULRF by following Mills. Mills has assumed that the production function of housing services is of a Cobb-Douglas type:

$$Xs(u) = AL(u)^{\alpha}K(u)^{1-\alpha} \qquad 0 < \alpha < 1 \quad \dots\dots\dots\dots (1)$$

where Xs (u) = the production of housing services at distance u from the CBD,

L (u) = input of land,
K (u) = input of capital,

The marginal product of land and capital are defined respectively

$$MPL(u) = AL(u)^{\alpha-1}K(u)^{1-\alpha} = \alpha Xs(u)/L(u),$$
$$MPK(u) = (1-\alpha)AL(u)^{\alpha}K(u)^{-\alpha} = (1-\alpha)Xs(u)/K(u).$$

The price of housing services at u, the rewards to land and capital are given by P (u), R (u), and r respectively.[3] For subjective equilibrium of the producer,

$$\alpha P(u)Xs(u)/L(u) = R(u), \quad \dots\dots\dots\dots (2)$$
$$(1-\alpha)P(u)Xs(u)/K(u) = r \quad \dots\dots\dots\dots (3)$$

hold. Therefore we get the relationship between the price of housing services and the factor prices by substituting (2) and (3) into (1).

$$Xs(u) = A\left[\frac{\alpha P(u)Xs(u)}{R(u)}\right]^{\alpha} \cdot \left[\frac{(1-\alpha)P(u)Xs(u)}{r}\right]^{1-\alpha},$$

We obtain the factor price frontier curve from above equation.

$$P(u) = [A\alpha^{\alpha}(1-\alpha)^{1-\alpha}]^{-1} \cdot R(u)^{\alpha}r^{1-\alpha}. \quad \dots\dots\dots\dots (4)$$

Differentiating (4) with respect to u, we get price profile of housing services.

$$\frac{dP(u)}{du} = A^{-1}\left[\frac{r}{1-\alpha}\right]^{1-\alpha} R(u)^{-(1-\alpha)}\frac{dR(u)}{du} \quad \dots\dots\dots\dots (5)$$

Mills has assumed that consumers have the identical utility functions and incomes and that they commute to the CBD. The subjective equilibrium condition for a location as well as the market equilibrium condition in this city is the following[4]

$$\frac{dP(u)}{du} x_D(u) + t = 0, \quad \cdots\cdots \quad (6)$$

where t stands for commuting cost per two unit distance. Mills has also assumed the demand function for housing services as a power function of income W and price such as

$$x_D(u) = BW^{\theta_1} P(u)^{\theta_2} \quad \cdots\cdots \quad (7)$$

where $\theta_1 > 0$, $\theta_2 < 0$.

As the level of income is to be common among the consumers, an aggregate demand for housing services at u is given

$$X_D(u) = N(u) \cdot x_D(u) \quad \cdots\cdots \quad (8)$$

where N (u) = number of consumers located at distance u. However, if we assume N (u) as an exogenous variable to the model, X_D is written as

$$X_D(u) = BW^{\theta_1} P(u)^{\theta_2}. \quad \cdots\cdots \quad (9)$$

if supply and demand of housing are equated

$$X_D(u) = Xs(u)$$

holds. Substituting (7) into (6), we get

$$\frac{dP(u)}{du} BW^{\theta_1} P(u)^{\theta_2} + t = 0. \quad \cdots\cdots \quad (10)$$

Substituting (4) and (5) into (10), we get the following equation.

$$A^{-1}\left(\frac{\alpha r}{1-\alpha}\right)^{1-\alpha} R(u)^{-(1-\alpha)} \frac{dR(u)}{du} BW^{\theta_1}$$

$$\left\{[A\alpha^{\alpha}(1-\alpha)^{1-\alpha}]^{-1} R(u)^{\alpha} r^{1-\alpha}\right\}^{\theta_2} + t = 0$$

Therefore, the relation between land rent R (u) and distance u is the following.

$$E^{-1} R(u)^{\beta-1} \frac{dR(u)}{du} + t = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

where $E^{-1} = \alpha B W^{\theta_1} [A\alpha^{\alpha}(1-\alpha)^{1-\alpha}]^{-(1+\theta_2)} r^{(1-\alpha)\cdot(1-\theta_2)}$

$\beta = \alpha\ (1+\theta_2)$

Equation (11), which is a differential equation of first order can be solved as the following forms by utilizing the bordered condition. We will call equation (12) and (13) the equilibrium land rent function.

$$R(u) = [\overline{R}^{\beta} + \beta t E(\overline{u}-u)]\ 1/\beta \quad \text{if } \beta \neq 0 \quad \ldots\ldots\ldots\ldots \quad (12)$$

and

$$R(u) = \overline{R}e^{tE(\overline{u}-u)} \quad \text{if } \beta = 0 \quad \ldots\ldots\ldots\ldots \quad (13)$$

where $\overline{u}$ = the distance from CBD to the edge of the urban area,

$R(\overline{u}) = \overline{R}$

$\overline{R}$ = rent on non-urban use of land.

Equation (13) is known as NEULRF which demonstrates that urban rents decline exponentially with distance.

III The Generalized Rent Function

Following the procedure developed by Kau and Lee and Kau and Sirmans[(5)], (12) can be written as

$$\frac{R(u)^{\beta}-1}{\beta} = \frac{\overline{R}^{\beta}-1}{\beta} + tE\ (\overline{u}-u), \quad \text{if } \beta \neq 0 \quad \ldots\ldots\ldots\ldots \quad (14)$$

Since there are only two observable variables in (14), i.e., land rent R (u) and distance from the CBD u, it can be written as

$$\frac{R(u)^{\beta}-1}{\beta} = R_0 - \gamma u \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

where

$$R_0 = \frac{\overline{R}^{\beta}-1}{\beta} + \gamma\overline{u} \qquad \text{and } \gamma = tE > 0$$

If β approaches zero, then (15) becomes

$$\log R(u) = \log \overline{R} - \gamma u, \qquad (16)$$

which corresponds to (13). Therefore NEULRF is a special case of the generalized functional form (13). Equation (12) and (13) belong to the first case of transformation of dependent variable that Box and Cox[6] have introduced to determine the true functional form. In general, an additive stochastic term can be introduced into (15)

$$\frac{R_i(u)^{\lambda} - 1}{\lambda} = R_0 - \gamma u_i + \epsilon_i, \qquad (17)$$

where $\lambda = \beta$ and we assume

$$\epsilon_i \sim N(0, \sigma^2).$$

In accordance with the maximum likelihood method, Box and Cox, have derived a maximum logarithmic likelihood for determining the functional form parameter. Following Box and Cox, we write equation (17) as

$$\vec{R}^{(\lambda)} = U\vec{\theta} + \vec{\epsilon} \text{ and } \vec{\epsilon} \sim N(\vec{0}, \vec{\sigma}^2) \qquad (18)$$

where $\vec{R}^{(\lambda)}$ is the column vector of the transformed observations, U is a known matrix and $\vec{\theta}$ is a vector of unknown parameters associated with the transformed observations. In our model, those elements are

$$\vec{R}^{(\lambda)} = \begin{bmatrix} \frac{R_1^{\lambda} - 1}{\lambda} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \frac{R_N^{\lambda} - 1}{\lambda} \end{bmatrix}, \quad U = \begin{bmatrix} 1 & u_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & u_N \end{bmatrix}, \quad \vec{\theta} = \begin{bmatrix} R_0 \\ -\gamma \end{bmatrix} \text{ and } \vec{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_N \end{bmatrix}$$

Since the expectation of $\vec{R}^{(\lambda)}$

$$E(\vec{R}^{(\lambda)}) = U\,\vec{\theta}\,,$$

then the likelihood in relation to the original observations $\vec{R}' = (R_1, \ldots\ldots, R)$ is

$$\left(\frac{1}{2\pi\sigma^2}\right)^{\frac{N}{2}} \text{exp.} \left\{- \frac{(\vec{R}^{(\lambda)} - U\vec{\theta})'(\vec{R}^{(\lambda)} - U\vec{\theta})}{2\,\sigma^2}\right\} J\ (\lambda\,;\vec{R}), \quad \ldots \quad (19)$$

where J is the Jacobian of the transformation from the variables $R_i^{(\lambda)}$ to R_i. Thus

$$J = \prod_{i=1}^{N} \left| \frac{dR_i^{(\lambda)}}{dR_i} \right| = \prod_{i=1}^{N} R_i^{\lambda-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

To find the maximum-likelihood estimates, we first take the likelihood for given λ. Under the normality assumption of the error term, the maximum-likelihood method is equivalent to the ordinary least-squares method. Therefore the maximum likelihood estimates of the θ's are the least-squares estimates for the dependent variable $R^{(\lambda)}$ are the estimates of σ^2, denoted for fixed by $\hat{\sigma}^2(\lambda)$, is

$$\hat{\sigma}^2(\lambda) = \vec{R}^{(\lambda)}\,Ur\vec{R}^{(\lambda)} / N - K - 1 = S(\lambda)/N - K - 1 \quad \ldots\ldots\ldots \quad (21)$$

where K is a number of independent variables and Ur is an idempotent matrix defined when U is of full rank:

$$Ur = I - U(U'U)^{-1}U'. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

S (λ) is the residual sum of squares in the analysis of variance of $R^{(\lambda)}$. Thus the maximized logarithmic likelihood for given λ is

$$\mathcal{L}\max(\lambda) = -\frac{N}{2}\log\ (2\pi\sigma^2(\lambda)) - \frac{N}{2} + \log J. \ldots\ldots\ldots\ldots \quad (23)$$

Substituting (20) into (23) and excluding constants, (23) becomes

$$\mathcal{L}\max(\lambda) = -\frac{N}{2}\log\sigma^2(\lambda) + (\lambda - 1)\sum_{i=1}^{N}\log R_i \quad \ldots\ldots\ldots \quad (24)$$

By this criterion we can find the optimum value of λ, denoted by $\hat{\lambda}$; which maximized the likelihood among different values of λ. Using the likelihood-ratio test, an approximate $(1-\alpha) \cdot 100\%$ confidence region for $\hat{\lambda}$ can be obtained from

$$\mathcal{L}\max(\hat{\lambda}) - \mathcal{L}\max(\lambda) < \frac{1}{2}\chi^2_1(\alpha) \quad \ldots\ldots\ldots\ldots\ldots \quad (25)$$

For example, an approximate 99% confidence region for λ is

$$\mathcal{L}\max(\hat{\lambda}) - \mathcal{L}\max(\lambda) < \frac{1}{2}\chi^2_1\ (0.01) = 3.35 \quad \ldots\ldots\ldots \quad (26)$$

The optimum point estimate, $\hat{\lambda}$, provides an estimate of β in (14).

IV Empirical Estimation

To examine the function form of the urban land rent in Tokyo, we apply the model and technique introduced in Section 3 and 4.

(A) Data

The data for this analysis are collected from the Public Announcement on Land Values[7] in 1972, 1976, and 1979. We select the standard lands which are located within 1 kilometer from the nearest railroad or subway station[8], in Tokyo Metropolitan Special Administrative Districts except Chūou, Koutou, Sumida, Adachi, Katsushika, and Edogawa. We take the railroad and/or subway stations below for representation of the CBD in Tokyo. Those stations are Tokyo, Ōtemachi, Yūrakuchō, and Hibiya.

To calculate time distance from the standard land to the CBD, we add up time distance from standard land to the nearest railroad or subway station and the shortest time distance from the nearest station to the nearest CBD station. If there are changes of train to commute the CBD, we add 5 minutes per each change.

We apply the following relation (27) between land rent and land value to obtain land rent.

$$R = P \cdot i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (27)$$

where

R = land rent per month

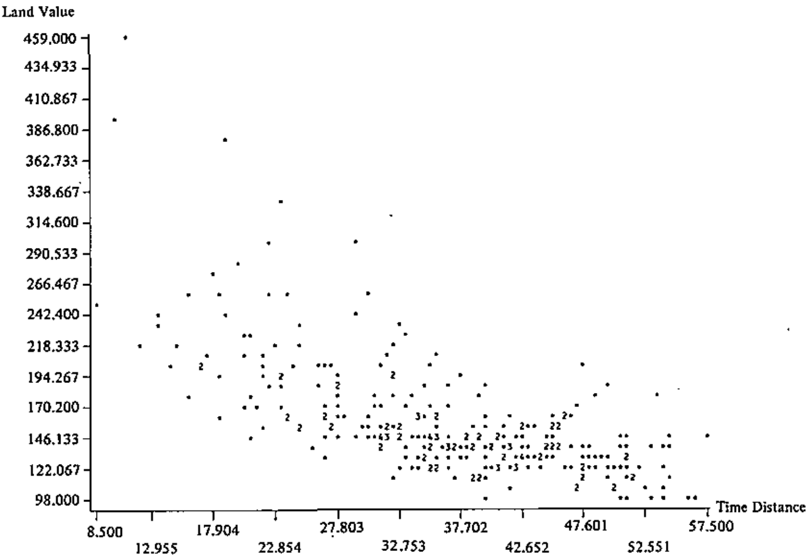
P = land value

i = interest rate per month.

We use $i = 0.604(\%)$ which is an interest rate of approximately 7.5 percent per year. Land values are thousand yen per one square meter, and so are land rents per month.

In Figure 1, we show a scattergram with land value and time distance in 1976.

Figure 1. Scattergram with Land Value and Time Distance (1976)



(B) Result of the Estimation

To find the functional form of the urban land rent in Tokyo, thirty-one regressions are estimated for each year. That is we give thirty-one values of β in the left hand side of (14) between -1.5 and 1.5 at intervals of 0.1. Note that when β takes a zero value, (14) becomes the negative exponential function (16) and when β takes 1, (14) becomes the linear function.

The true functional form is determined by choosing which maximized the likelihood (24). The maximum likelihood estimate of (17) are presented in Table 1 together with the estimated negative exponential function.

In recent study of Yamada et al., they apply a negative exponential land value function of the distance to the case of Tokyo in 1968 and obtain following estimated equation.[9]

$$P(u) = \underset{(39.48)}{57.591} \; \exp.^{\underset{(8.71)}{-0.00245}u} \qquad \bar{R}^2 = 0.6632$$

Their study is one of a few studies available which can be compared with our study. Power parameter of theirs is a little greater than it of ours. We may conjecture that the difference of the magnitude of power parameter attribute to the difference of the size of urban area considered. In their study, the urban area stands for Tokyo, Kanagawa, Saitama, Chiba, and southern part of Ibaraki prefecture.

Table 1. Urban Land Rent Function of Tokyo: 1972 to 1979

Year	Regression	β-value	Constant	Distance	$\bar{R}^2$	$(\bar{R}^2)$[a.]
1972 (188)[b.]	NE	0.	0.1195 (2.87)[c.]	−0.01779 (15.92)	.577	.571
	ML	−0.8	0.2674 (4.37)	−0.02611 (15.19)	.576	.575
1976 (297)	NE	0.	0.5498 (15.38)	−0.01666 (17.53)	.510	.491
	ML	−1.1	0.5047 (14.45)	−0.01632 (17.58)	.512	.511
1979 (308)	NE	0.	0.7553 (18.59)	−0.01719 (16.15)	.460	.450
	ML	−1.1	0.5967 (18.11)	−0.01385 (16.04)	.457	.463

a. We will discuss this value later.

b. Number of observations are in parenthesis below each year.

c. t-values are in parenthesis below coefficient.

Since β is defined as $\beta = \alpha(1 + \theta_2)$ in which α is defined in the production function (1) and θ_2 is defined in the demand function (9), the price elasticity of housing services, θ_2, can be obtained from α and θ_2. If we assume that α statisfies

$$0.3 < \alpha < 0.7$$

then θ_2 can be calculated from α and the estimated value of β. We show the result in Table 2.

Table 2. Price Elasticity of Housing Services

	$\beta = -0.8$ (in 1972)	$\beta = -1.1$ (in 1976 and 1979)
α	θ_2	θ_2
0.3	-3.67	-4.66
0.4	-3.00	-3.75
0.5	-2.60	-3.20
0.6	-2.33	-2.83
0.7	-2.14	-2.57

We can conclude that the price elasticity of housing services is elastic comparing the results which Kau and Sirmans (4) have presented. Their results are summarized below in Table 3.

Table 3. Urban Land Rent Function of Chicago: 1836 to 1928[a]

Year	Regression	β-value	Constant	Distance	$\bar{R}^2$	Price elasticity[b]
1836 (208)	NE	0.	5.632 (44.832)	−0.403 (27.168)	.781	
	ML	−0.25	3.247 (62.550)	−0.197 (36.800)	.834	−2.25
1857 (211)	NE	0.	8.748 (70.886)	−0.513 (35.742)	.858	
	ML	−0.07	6.791 (79.715)	−0.365 (36.800)	.866	−1.35
1873 (207)	NE	0.	9.980 (71.655)	−0.344 (21.011)	.682	
	ML	−0.10	6.386 (98.374)	−0.161 (21.011)	.684	−1.50
1892 (173)	NE	0.	10.043 (52.558)	−0.246 (11.169)	.418	
	ML	−0.09	6.611 (74.400)	−0.134 (12.678)	.401	−1.40
1910 (123)	NE[c]	0.	10.584 (52.018)	−0.319 (12.678)	.566	
1928 (139)	NE	0.	11.736 (72.390)	−0.220 (11.735)	.497	

a. The source is Kau and Sirmans Table 1 and Table 2.

b. The point estimate of the elasticity of demand for housing services is based on $\alpha = 0.2$.

c. The negative exponential equation is the maximum likelihood estimate for 1910 and 1928.

In Table 4, we show the best parameter of the transformation in column (1), likelihood when λ takes $\hat{\lambda}$ and zero in column (2) and (3) respectively and the difference between (2) and (3) in column (4) for each year. The hypothesis that $\hat{\lambda}$ equals to zero can be rejected at the 99 percent significance level from the likelihood ratio test (26). Therefore NEULRF statistically proves to be incorrect for our cases.

Table 4. Likelihood of Estimated Equations

	(1)	(2)	(3)	(4)
Year	$\hat{\lambda}$	$L(\hat{\lambda})$	$L(0)$	$L(\hat{\lambda}) - L(0)$
1972	-0.8	477.377	470.899	6.478
1976	-1.1	568.031	543.905	24.114
1979	-1.1	509.622	486.079	23.543

However, the coefficient of determination, R^2, which is one measure for fitting of the equation shows that there is a very little gain by taking the generalized functional form in 1972 and even loss in 1972 and 1979.

To reinterpret these facts, we introduce the following idea and define another value to evaluate the fitting of the equation. In the theoretical model, the rent function is

$$R(u) = [\overline{R}^{\beta} + \beta tE(\overline{u} - u)]^{\frac{1}{\beta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

The stochastic model of (28) is following.

$$R(u) = [\overline{R}^{\beta} + \beta tE(\overline{u} - u)]^{\frac{1}{\beta}} + \epsilon_1, \ \epsilon_1 \sim N(0, \sigma^2) \quad \ldots\ldots \quad (29)$$

However, we cannot estimate (28) directly. Therefore we transform (28) and get (30)

$$\frac{R(u) - 1}{\beta} = \frac{\overline{R}^{\beta} - 1}{\beta} + tE(\overline{u} - u). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (30)$$

The stochastic model of (30) is as follows

$$\frac{R(u)-1}{\beta} = \frac{\overline{R}^{\beta}-1}{\beta} + tE(\overline{u}-u) + \epsilon_2, \; \epsilon_2 \sim N(0,\sigma^2). \; \ldots \quad (31)$$

The relationship between (28) and (30) is consistent, in other words it is reversible. We may call (28) the original form and (30) the reduced form. However, if we take the model (31), the model (29) is inconsistent with (31). Following the Box-Cox technique, we have taken (31) for estimation. Since ϵ_2 is assumed to belong to a normal distribution, we have estimated above equation by the ordinary least square method for given β and obtain statistics such as the likelihood and the coefficient of determination (R^2). The R^2 defined in (31) is a correlation coefficient between $\frac{R^{\beta}(u)-1}{\beta}$ and the systematic parts of (31). On the other hand, the R^2 defined in the negative exponential and rent function is a correlation coefficient between R (u) and the systematic parts. Therefore it is not suitable to compare the R^2 of the two equations when we evaluate the fitting of the land rent function. For comparing the fitting, we introduce (32) instead of (31).

$$R(u) = \overline{R}^{\beta} + tE(\overline{u}-u)^{\frac{1}{\beta}} + \epsilon_3, \; \epsilon_3 \; \chi \; N \; \ldots \quad (32)$$

(32) can be interpreted a proxy of (29). If we apply the estimated coefficients of (31) into (32), we can define the coefficient of determination in (32) because the definition of it itself does not assume the normality of the error term. In the last column in Table 1, we show the coefficient of the determination of (32). These values indicate that the generalized land rent function improves the fitting a little.

Applying the estimated coefficients, the generalized land rent function can be expressed as

$$R(u) = [\beta R_0 + 1 - \beta\gamma u]^{\frac{1}{\beta}} \; \ldots \quad (33)$$

Taking the derivative of (33) with respect to u, and rearranging terms, the generalized rent gradient is,

$$\frac{\partial R}{\partial u} = -\gamma \frac{R}{R^{\beta}} \qquad (34)$$

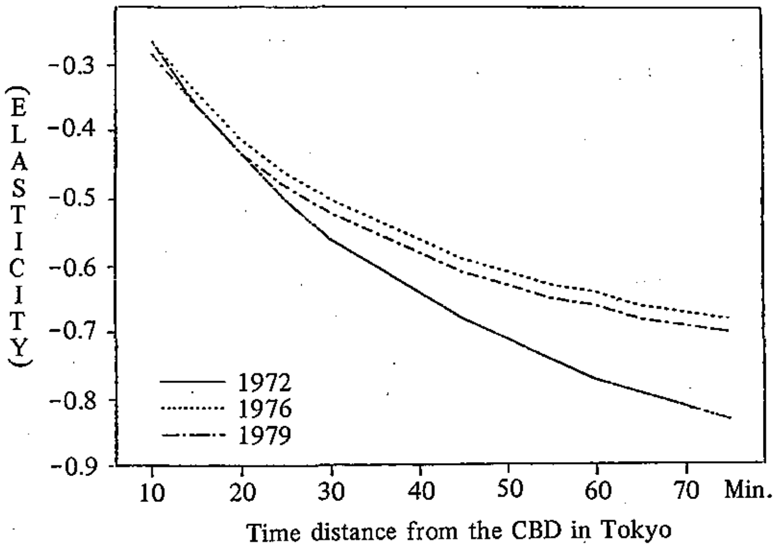
The elasticity of rent with respect to distance is

$$\eta = \frac{\partial R}{\partial u} \frac{u}{R} = \frac{-\gamma u}{R^{\beta}} \qquad (35)$$

If NEULRF is assumed, land rent gradient is constant, in other words, land rent is decreasing with a constant rate with respect to distance. Though the generalized land rent function assumed, land rent gradient is not constant.

We show the elasticity of the rent function we have estimated in Figure 2 below.

Figure 2. Elasticity of Rent with Respect to Distance



V Summary and Conclusions

In this paper, we have provided empirical evidences on the functional form of the relationship between land rents and time distance from the CBD by using the Japan Land Agency's data on Tokyo in 1972, 1976 and 1979. Utilizing the Box and Cox transformation technique, we have found that NEULRF is not correct functional form and we have estimated the generalized urban land rent function.

Though the generalized urban land rent function is also based upon the same simple assumptions by which we have derived NEULRF. The assumptions are identical utility function and income for all households within the monocentric city together with simple functional forms of demand and supply of housing services. To understand the structure of the urban land rents in actual cities such as Tokyo, further research is necessary.

(Oct. 23, 1980)

Notes

(1) Muth, R. F., *Cities and Housing,* University of Chicago Press, Chicago 1969: Mills, E. S., *Urban Economics,* Scott, Foresman and Company, Glenview, 1972.

(2) Kau, J. B. and Sirmans, C. F., "Urban land value functions and price elasticity of demand for housing", *Journal of Urban Economics* 6, pp. 112-121, 1979.

(3) The rewards to capital are assumed to be common with in the urban area considered.

(4) Equation (6) claims that households are unable to increase utility by moving their location because the change in the cost of housing services from a move is just offset by the change in commuting cost.

(5) Kau, J. B. and Lee, C. F., "Functional form, density gradient and price elasticity of demand for housing", *Urban Studies* 13, pp. 193-200, 1976: Kau and Sirmans, *op. cit.*

(6) Box, G. E. P. and Cox. D. R., "An analysis of transformations", *Journal of Royal Statistical Society* 26 (Series B), pp. 211-243, 1964.

(7) The Japan Land Agency publishes the Public Announcement on Land Values annually. In 1979, the report contains of land values of

16,480 standard lands together with their location, usage and other informations which help adequate transaction of land.

(8) If we take the standard land locations outer 1 kilometer from the station, we have to consider other transportation such as bus, and it seems impossible to compare time distance of the standard lands.

(9) Yamada, H., et al., "An Econometric Analysis on Housing Market in Tokyo Megalopolitan Area" (in Japanese), *Japan Economic Planning Agency Study Series* No. 31., Economic Planning Agency, Tokyo, 1976.

一般化地代関数の実証研究

〈要　　約〉

大河原　　透

この論文ではミュース，ミルズらの都市経済モデルから導出される都市住宅地の地代関数の推定を東京都区部のデータを用いて行なった。理論的地代関数は本文(12),(13)式で示されるような負の指数関数属であるが，推定上の容易さという観点より(12)式の推定が行なわれることが多く，(13)式の推定は日本では全く行なわれていなかったのが現状である。しかし，(12)式は住宅サービスへの需要関数の価格弾性値が1であるという非常に強い仮定から導出されている。価格弾性値に先験的な仮定を置くことなしに(13)式をボックスとコックスの変数変換理論を用い推定を試みたところ，(12)式は99パーセント以上の統計的有意さで棄却され，(13)式の一般化された負の指数関数の地代関数が得られた。このことは地代関数の推定を行なうに際し，一般化された地代関数を推定すべきことを強く要請しているといえる。また一般化された地代関数では，住宅サービスへの需要関数の価格弾性値の他，距離に関しての地代の弾性値の情報も得られ有用である。しかしながら，この研究で推定された価格弾性値は一般に信じられている値よりもかなり大きく，それがまた東京都区部というかなり特殊な都市の土地事情を反映しているものなのかもしれない。