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Expectations, learning dynamics and the stabilization policy in an overlapping generations model

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Abstract

The aim of this paper is to explore whether a least squares learning is able to lead the dynamics of an economic system to perfect foresight equilibrium, and whether a stabilization policy can stabilize unstable least squares learning dynamics in a monetary overlapping generations model. We show that least squares learning dynamics are unstable if agents have strong wealth effects which dominate intertemporal substitution effects, and overreactionary expectations. Therefore, a least squares learning is a source of endogenous learning which drives economic fluctuations, and moreover, misperceptions doesn't vanish in the least squares learning dynamics. Then we illustrate that a stabilization policy (the increase of the growth rate of money supply through lump sum transfers) can stabilize any unstable learning dynamics when the least squares learning cannot lead to the perfect foresight equilibrium.

Key words : adaptive (overreactionary) expectations, least squares learning, stabilization policy, and chaos.

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1 Introduction

Self-fulfilling expectations are often viewed as a long run concept, where agents have already learned completely the law of motion governing the system in which they operate. One should expect the agents to make significant forecasting mistakes for quite some time while they attempt to learn the dynamic laws of the signals they receive. It is thus quite natural that the first question we shall address is how expectations and learning take place, and more importantly, whether expectations and learning will lead towards perfect foresight equilibrium. If the temporary equilibrium dynamics with learning (*learning dynamics*) don't converge to perfect foresight equilibrium in the long run, then the corresponding efficiency losses will appear. The next question we must consider is whether stabilization policies implemented by the government to get rid of efficient losses, can lead to stabilizing the unstable learning dynamics. Although a considerable number of studies have been made on these questions over the past few years, there is little agreement on how the economic system is influenced by expectations and learning[1] .

The aim of this paper is to propose an answer to the two questions posed above. The basic model we shall use here is the simple 'Samuelsonian' model of overlapping generations[2] . We apply *adaptive (overreactionary) expectations* as an alternative expectations form to perfect foresight and we assume that agents estimate the unknown structural parameter,(error correction coefficient) by using a *least squares learning.* First, we demonstrate that (i) if agents have strong intertemporal substitution effects which dominate wealth effects, and/or weakly adaptive expectations, temporary equilibrium dynamics converge to perfect foresight equilibrium, and (ii) if agents

[1] On the recent works of expectations and learning mechanisms, see Day and Lin (1992), Grandmont and Laroque (1986, 1992), Guesnerie and Woodford (1992), Marcet and Sargent (1989), Benassy and Blad (1989), Evans and Honkapohja (1994), Bullard (1994) and Kaizoji (1995).

[2] Benhabib and Day (1982), and Grandmont (1985) demonstrate that complex endogenous cycles occur as intertemporal equilibrium phenomena Their studies demonstrate that persistent economic fluctuations generated by volatile forecasts are indeed compatible with individual optimization, self-fulfilling expectations and Walrasian market clearing provided that there are capital market imperfections. The outcome of these studies is that under increasingly plausible assumptions, endogenous fluctuations equilibria with self-fulfilling expectations do occur in such models, see Boldrin and Woodfords' review [Boldrin and Woodford (1990)].

have strong wealth effects which dominate intertemporal substitution effects, and/or strongly overreactionary expectations, temporary equilibrium dynamics are able to be chaotic. Then we illustrate that (iii) a least square learning for expectations leads to the perfect foresight equilibrium, provided that the endogenous fluctuations are caused by agents' overreactionary expectations, and (iv) on the contrary the least squares learning cannot lead to the perfect foresight equilibrium, provided that the endogenous fluctuations are caused by strong wealth effects. Finally, we show that (v) a stabilization policy (an increase of the growth rate of money supply) can stabilize unstable learning dynamics when the least squared learning cannot lead the economic system to the perfect foresight equilibrium.

2 The model

Consider an overlapping generations model where each generation lives two periods. The model involves one non-storable good and a single asset, money, that is employed for transferring wealth from one period to the next. For most of this paper, the money stock, M will be assumed to be constant over time. Agents live two periods and are identical (equivalently there is a single agent) in each generation. The agents' endowments of the good at each age $\tau = 1, 2$, are e_1 and e_2. The representative consumer is assumed to have the separable utility function,

$$U(c_1, c_2) = \log c_1 + c_2^{(1-\beta)}/(1-\beta), \tag{1}$$

where c_1 and c_2 denote consumption in the first and second periods of life respectively. As concerns the attitude to risk, this utility function is known as 'constant relative risk aversion' (CRRA), where the RRA coefficient is $-v_2''(c_2)/v_2'(c_2) = \beta$. A large β, which is greater than unity, means that an agent have a strong wealth effect which dominates an intertemporal substitution effect, and on the contrary a small β, which is less than unity, means that an agent have a weak wealth effect[3] .

We focus on the *Samuelson case.* Namely, we consider that the consumers hold positive money balances, (that is, technically this can be ensured by

[3] See the chapter 5 of Blanchard and Fischer (1985) for a full account of the relation between a wealth effect and β.

choosing the endowments appropriately.) Under the foregoing environment we can get the well-defined difference equation that represents the law of motion of the economic system

$$\mu_t = e_1\mu^e_{t+1}/[\mu^e_{t+1} + (\mu^e_{t+1} + e_2)^\beta] \equiv \chi(\mu^e_{t+1}). \tag{2}$$

where μ_t and μ^e_{t+1} denote the real balance and the real balance expected by the young agent, respectively. The steady states of (2) are 0 and μ^* where $e_1 = \mu^* + (\mu^* + e_2)^\beta$. It is proved easily that (2) has the unique stationary monetary equilibrium, μ^*.

2.1 Expectations

We have a dynamic system where the expectations determine the current variables through equation (2). In order to define the dynamics fully, we need to specify the expectations formation function. If agents do know at the beginning of the period t only the past values of the real balances, $(\mu_t, \mu_{t-1}, \ldots\ldots)$, but they don't know the current value of the real balance, then young agents have to forecast the future values of the real balance by using the past values of real balances at the beginning of the period t.

One way to expectations formation is to postulate that at each period, the real balance μ^e_{t+1} expected by young agents is a fixed function of the past values of the real balance. In this paper, we assume that the expectations are formed as follows[4] :

$$\mu^e_{t+1} = (1-\alpha)\mu^e_t + \alpha\mu_{t-1}, \tag{3}$$

where α is a constant of proportionality called the error correction coefficient. Substituting (3) into (2), we get

$$\mu^e_{t+1} = (1-\alpha)\mu^e_t + \alpha\chi(\mu^e_t) \equiv F_\alpha(\mu^e_t). \tag{4}$$

[4] If the infinite sum converges, then The expectations formation function (3) is the same as the weighted average of past values :

$$\mu^e_{t+1} = \alpha\sum_{T=1}^{\infty}(1-\alpha)^{T-1}\mu_{t-T}.$$

Let us name the case of $0 < \alpha < 1$ the adaptive case, and the case of $1 \leq \alpha$ the overreactionary case.

The difference equation (4) defines a *forward temporary equilibrium dynamics with adaptive (overreactionary) expectations.*

The map $F_\alpha(\mu^e)$ has two steady states, the perfect foresight equilibrium, μ^* and the no-trade equilibrium, 0. A steady state of the map $\chi(\mu^e)$ is equal to a steady state of the map $F_\alpha(\mu^e)$. The proof of this statement can be given immediately. The perfect foresight equilibrium satisfies the equation $\mu^* - \chi(\mu^*) = 0$. Then the temporary equilibrium dynamics (4) also become the steady state $\mu^e_{t+1} = \mu^e_t$. Thus the above statement is proved. That means that a *stationary temporary equilibrium with adaptive (or overreactionary) expectations* is equal to a *perfect foresight equilibrium.* We will investigate the properties of the forward temporary equilibrium dynamics (4) in Section 3 below.

3 Temporary equilibrium dynamics with adaptive expectations

In the present section we study the dynamic propensities of temporary equilibrium dynamics which are generated by the equation (4).

3.1 Stability of the perfect foresight equilibrium

We begin the analysis by demonstrating the condition of the local stability of the perfect foresight equilibrium μ^*.

Proposition 1 : If $\chi'(\mu^*) = (\mu^*/e_1)[1 + \beta(e_1 - \mu^*)/(\mu^* + e_2)] < 2/\alpha$, where μ^* is the unique perfect foresight equilibrium, then the perfect foresight equilibrium μ^* is locally stable.

Proof of Proposition 1 : It follows that under the above condition the derivative of $F(\mu^e)$ at the perfect foresight equilibrium μ^* is less than 1. Namely, $F'_\alpha = 1 + \alpha[\chi'(\mu^*) - 1] < -1$. The perfect foresight equilibrium μ^* is thus locally stable. (Q.E.D.)

Proposition 1 demonstrates that if the error correction coefficient α and/or RRA coefficient β are small enough, then the temporary equilibrium dynamics converge to the stationary monetary equilibrium. In other words, if an agent has a very weakly adaptive expectation and/or a weak wealth effect, the perfect foresight equilibrium has stability.

3.2 Chaotic dynamics

In this section we will look at how the dynamic propensities of the temporary equilibrium dynamics (4) change as β and α become large. We will demonstrate a sufficient condition for the temporary equilibrium dynamics to lead to *chaos* below.

We begin to study the effects of the changes of RRA coefficient β. For simplicity of analysis we assume $\alpha = 1$. In this case the expectation formation function become $\mu^e_{t+1} = \mu_{t-1}$. Under this condition Grandmont (1985) and Kelsey (1988) prove the following proposition[5] ;

Proposition 2 : For all sufficiently large β, the map $F_\alpha(\mu^e)$ has a 3-cycle provided $e_2 < 1$ and $e_1 + e_2 > 1/e_2$.

Proof of Proposition 2 : See the proposition of Grandmont (1985) and the proposition 3.3 of Kelsey (1988).

Li and Yorke (1975) prove that the existence of 3-cycles implies chaos. Therefore the temporary equilibrium dynamics (4) become chaotic for any sufficiently large β.

Figure 1 is the bifurcation diagram of μ^e_t where the RRA coefficient β varies smoothly from 5 to 30 under the specification $\alpha = 1$, $e_1 = 10$ and $e_2 = 0.95$. From Figure 1 we can know that μ^* begins to be unstable where the value of β is equal to 5.754. (We can calculate the value of β by using the condition of Proposition 1.) Figure 1 also shows chaos to occur through a sequence of period doublings which is one route to chaos. Thus, Figure 1 illustrates the results of Proposition 2.

Then we study the effects of the changes of the error correction coefficient α to the temporary equilibrium dynamics.

Proposition 3 : Assume that $\alpha > 2/\chi'(\mu^*)$. Then there exists α such that the temporary equilibrium dynamics (4) are chaotic in the sense of Li-Yorke.

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[5] Benhabib and Day (1982) demonstrates the sufficient conditions for Li-Yorke chaos in Gale economy.

Proof of Proposition 3[6] : The map χ and F satisfy the following ;

$$\begin{array}{ll} \chi(0)=0, & \chi(\mu^*)=\mu^*, \\ F_\alpha > \mu^e & \text{for } 0<\mu^e<\mu^* \\ F_\alpha < \mu^e & \text{for } \mu^*<\mu^e<+\infty. \end{array}$$

Figure 2 (a) and Figure 2 (b) show the map χ and the corresponding map F_α. $\bar{\mu}^e$ denotes the value of μ^e which maximizes the map F_α, and μ_c^e denotes the value of μ^e where $F_\alpha(\mu_c^e)=0$. The map F_α in Figure 2 (b) is more hump than the map $\chi(\mu^e)$ in Figure 2 (a). It means that α is greater than 1. Under the above conditions the maximum of F_α increases as α becomes large, and the maximum of F_α approaches to $+\infty$, as α approach to $+\infty$. On the other hand the value of μ_c^e, decreases as α becomes large, and μ_c^e approaches to μ^*, as α approaches to $+\infty$. From the continuity of the map F_α with respect to α there exists α such that the maximum of $F_\alpha(\mu^e)$ is equal to μ_c^e. Therefore it follows that there exist α such that F_α satisfies the so-called overshoot condition :

$$F_\alpha^2(\bar{\mu}^e) < F_\alpha^3(\bar{\mu}^e) \leq \bar{\mu}^e < F_\alpha(\bar{\mu}^e). \tag{5}$$

(Q.E.D.)

Figure 3 is the bifurcation diagram of μ_t^e where the error correction coefficient α varies smoothly from 0.9 to 1.3 under the specification $\beta = 5.754$, $e_1 = 10$ and $e_2 = 0.95$. Figure 3 shows chaos to occur through a sequence of period doublings as α becomes large. Finally Figure 4 summarizes the propensities of the temporary equilibrium dynamics with adaptive (overreactionary) expectations. These figures demonstrate that cyclic and chaotic adaptive learning dynamics are caused by any large β and/or any large α.

In other words, if agents have strong wealth effects, complex endogenous fluctuations occurs, and the complexity of the learning dynamics is amplified by overreactionary expectations.

4 Least squares learning dynamics

In the foregoing section we assume that the expectation formation function is the fixed function (3) over the time of past values of μ_t. However, it seems

[6] For the proof of Proposition 3, see also Kaizoji (1995).

natural to suppose that at the beginning of the period t, agents get some estimate α_t of the error correction coefficient α by applying a given statistical procedure to past observations, $\alpha_t = h(\mu_{t-1}, \mu_{t-2}, \ldots\ldots\ldots)$. Let us now attempt to extend this consideration into the adaptive learning dynamics. It seems that a least squares learning rule is one of most natural statistical methods that agents get some estimate of α. We assume the following least squared learning rule of α,

$$\min \sum_{T=1}^{T-2} [\mu_{t-T} - \mu_{t-T}^e]^2. \tag{6}$$

where

$$\mu_{t-T}^e = (1-\alpha)\mu_{t-T-1}^e + \alpha\chi(\mu_{t-T-1}^e). \tag{7}$$

Substituting (8) into (7) and solving the above minimization problem, we get

$$\alpha_t = \frac{\sum_{T=1}^{T-2}(\mu_{t-T} - \mu_{t-T-1}^e)(\mu_{t-T-2} - \mu_{t-T-1}^e)}{\sum_{T=1}^{T-2}(\mu_{t-T-2} - \mu_{t-T-1}^e)^2}. \tag{8}$$

Therefore the least squares learning dynamics are :

$$\mu_{t+1}^e = (1-\alpha_t)\mu_t^e + \alpha_t\chi(\mu_t^e). \tag{9}$$

It is difficult to analyze the least squares learning dynamics (9) mathematically because (9) is a multi-dimensional difference equation. Thus we show the dynamic properties of (9) through computer simulations. Figure 5 summarizes the propensities of the least squares learning dynamics (9). In Figure 5 α of the horizontal axis is that the initial values of α before agents begin least squares learning.

Comparing Figure 4 and Figure 5, we find that in the area D in Figure 5 least squares learning dynamics converge to the perfect foresight equilibrium μ^* while temporary equilibrium dynamics (4) are unstable. To look at this more concretely, we assume $\alpha_0 = 1.3$, $\beta = 5$, $e_1 = 10$, and $e_2 = 0.95$ where α_0 denotes the initial value of the error correction coefficient. Figure 6 shows both time paths of the temporary equilibrium dynamics (4) and the least squares learning dynamics (9) under this set of parameters. Time series of temporary equilibrium dynamics are chaotic (the area A in Figure 6). The point, a is the starting point of the least squares learning. After the agents do the learning of α according to (8), the value of α changes from 1.3 to about 0.88, and so the least squares learning dynamics (9) converge quickly towards the perfect foresight equilibrium μ^* (the area B in Figure 6). We also find that, in all values of (α_0, β) that we put computer simulations in practice, α_t is staying in the area of $0 < \alpha_t < 1$ after agents begin the least squares learning. Thus the least squares learning is effective against the stabilization of the chaotic dynamics which is caused by overractionary expectations $(\alpha_0 > 1)$.

On the other hand Figure 7 shows both time paths of the temporary equilibrium dynamics and the least squares learning dynamics under a large β. We assume $\alpha_0 = 1$, $\beta = 18.5$, $e_1 = 10$, and $e_2 = 0.95$. Under this set of parameters, time series of temporary equilibrium dynamics are chaotic (the area A in Figure 7). The point, a is the starting point of the least squares learning. After the agents do the learning of α according to (8), the value of α changes from 1 to about 0.77. Nevertheless the least squares learning dynamics (9) occur bounded irregular fluctuations (the area B in Figure 7). The reason is that a strong wealth effect, that is, a large β, give cause for endogenous fluctuations. In this case agents' misperceptions never vanish in the least learning dynamics (9) even in the long-run term.

5 Stabilization policy

It follows from the foregoing analyses that the unstable least squares learning dynamics are caused by the wealth effect which dominates an intertemporal substitution effect. Since the instability of learning dynamics means that young agents have forecasting mistakes of the real balances at the next period, it will reduce economic efficiency.

Therefore, it is an important question in this case whether any economic policy which the government implements can stabilize the unstable learning dynamics.

According to Grandmont (1986), let us assume in addition that there is a government. At period t, it gives a lump sum money subsidy S_t (to be interpreted as a lump sum tax if negative) to the old agent. We define the growth rates of the money supply as follows,

$$s_t = (M_{t-1} + S_t)/M_{t-1}, \tag{10}$$

where $M_{t-1} > 0$ designates the money stock at the outset of period t. Given $M_0 > 0$, the evolution of the money supply is then ruled by

$$M_t = M_{t-1}s_t > 0, \quad \text{given} \quad M_0 > 0, \tag{11}$$

A newborn consumer has to solve the following decision problem. Let $p > 0$ be the current money price of the good, and Δm^e lump sum subsidy that he expects at the present period for the next period. He must choose then his current and future consumption $c_1 \geq 0$, $c_2 \geq 0$, and his money demand $m \geq 0$ so as to maximize his utility function subject to

$$pc_1 + m = pe_1 \quad \text{and} \quad p^e c_2 = p^e e_2 + \Delta m^e. \tag{12}$$

We assume that the government informs correctly the young consumer of the quantities, Δm at the next period. Then the least squares learning dynamics are given by the following difference equations ;

$$\mu_{t+1}^e = (1-\alpha_t)\mu_t^e - \frac{\alpha_t e_1 \mu_t^e}{[\mu_t^e + s_t(e_2 + \mu_t^e)^\beta]}. \tag{13}$$

In this case the perfect foresight equilibrium is μ^* where the equality, $e_1 = \mu^* + s_t(\mu^* + e_2)^\beta$ holds. It follows from this equality that the change the growth rate of money supply s_t, (monetary policy) influences the set of stationary state magnitudes, so that it influences the least squares learning dynamics.

Figure 8 illustrates this stabilizing effect of the monetary policy. To analyze the effect of the monetary policy, we assume that s_t is a parameter s, and choose $\beta = 18.5$, $e_1 = 10$, $e_2 = 0.95$, $s = 1$ and $\alpha_0 = 1$ as the initial value. As we illustrate above, both of the temporary equilibrium dynamics (4) and the least squares learning dynamics (9) are unstable under this set of parameters. The time series of μ_t^e of the area A in Figure 8 are the temporary equilibrium dynamics, and the time series of μ_t^e of the area B are the least squares learning dynamics before the implementation of the monetary policy under $\beta = 18.5$, $e_1 = 10$, and $e_2 = 0.95$. Both time series of μ_t^e fluctuate irregularly. Then, we consider that the government increases the rate of growth of money supply from $s = 1$ to $s > 1$. In Figure 8 the point b is the starting point of implementation of the monetary policy. The perfect foresight equilibrium μ^* decreases and the graph of the map χ is pushed down by the monetary policy. Hence if the growth rate of money supply s is large enough, then the learning dynamics may converge to the perfect foresight equilibrium. The time series of μ_t^e of the area C in Figure 8 are the least squares learning dynamics after the implementation of the monetary policy under $\beta = 18.5$, $e_1 = 10$, $e_2 = 0.95$, and $s = 3.5$. After the implementation of the monetary policy, the learning dynamics converge to the new perfect foresight equilibrium. Figure 9 summarizes the stabilizing effects of the monetary policy. Figure 9 illustrates that any unstable least squares learning dynamics can be stabilized by the monetary policy.

6 Concluding Remarks

In conclusion, (i) if agents have strong wealth effects which dominate intertemporal substitution effects, then the least squares learning dynamics are unstable, and (ii) a stabilization policy (an increase of the rate of growth of money supply through lump sum transfers) can stabilize any unstable learning dynamics when the learning cannot lead to the perfect foresight equilibrium.

Our findings suggest that a least squares learning is able to be an independent source of endogenous learning which drives economic fluctuations, and moreover that significant forecasting mistakes may not vanish in the least squares learning dynamics. These misperceptions may reduce economic efficiency. Then it is important to stabilize the learning dynamics by government's intervention.

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Figure 1

Figure 1 is the bifurcation diagram of μ_t^e where the RRA coefficient β varies smoothly from 5 to 30 under the specification $\alpha = 1$, $e_1 = 10$ and $e_2 = 0.95$. From Figure 1 we can know that μ^* begins to be unstable where the value of β is equal to 5.754. (We can calculate the value of β by using the condition of Proposition 1.) Figure 1 also shows chaos to occur through a sequence of period doublings which is one route to chaos.

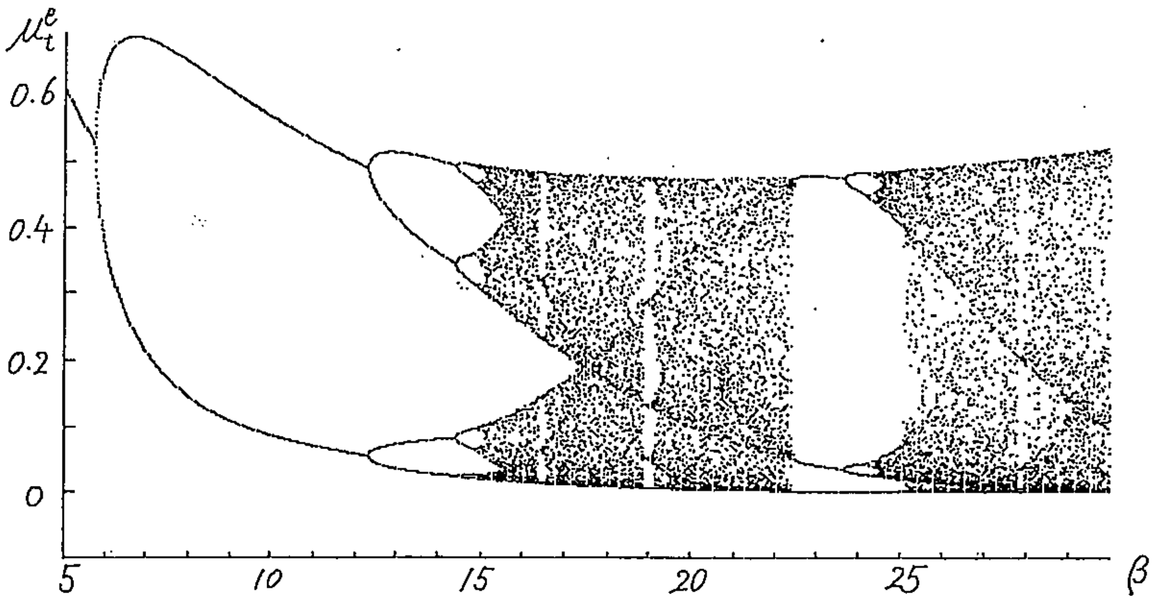


Figure 2 (a)

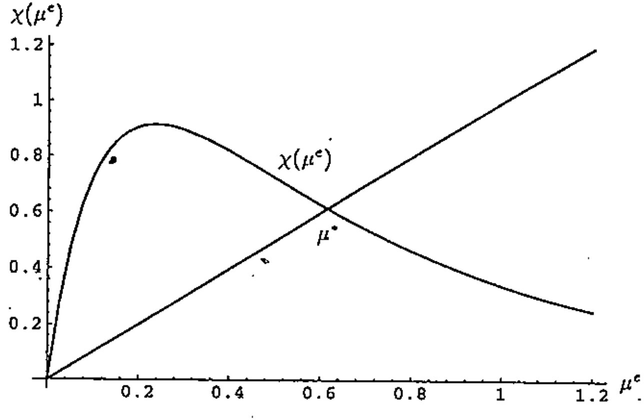


Figure 2 (b)

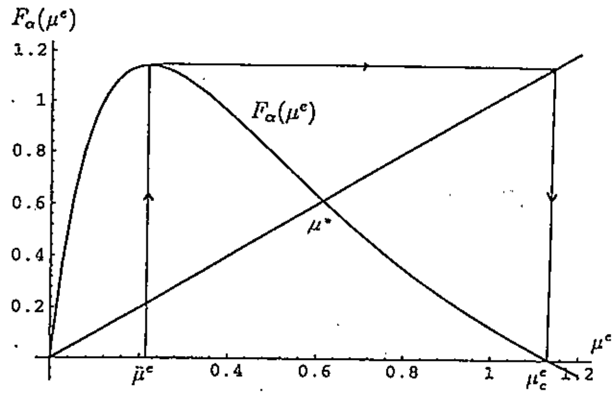
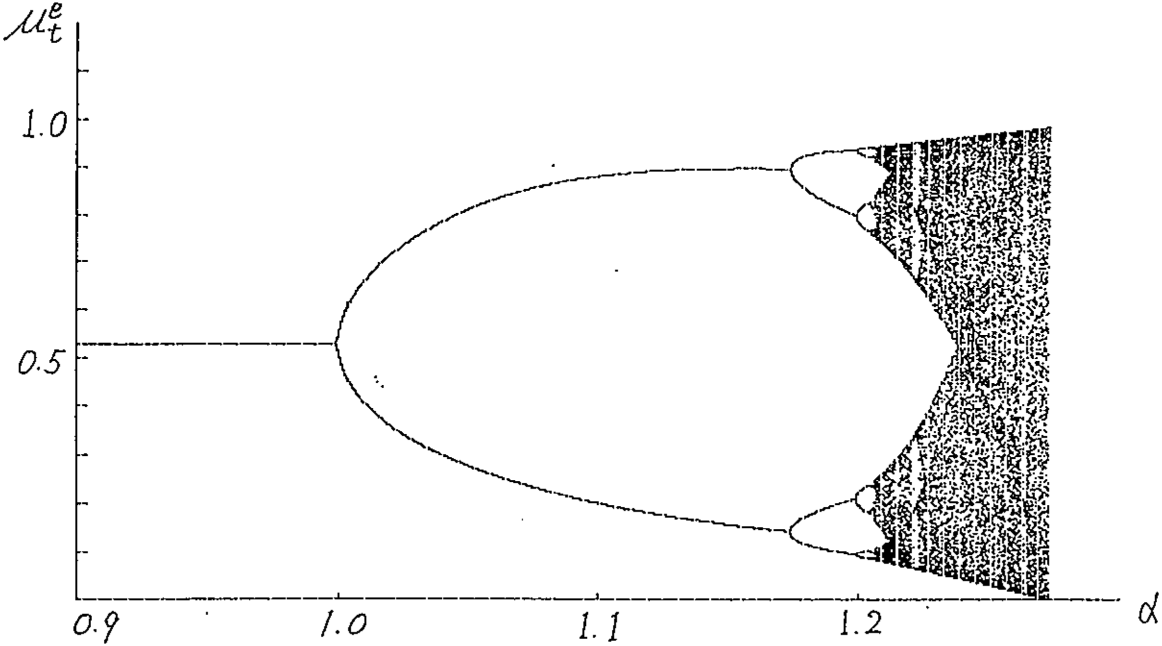


Figure 2 (a) Figure 2 (b)

Figure 2 (a) and Figure 2 (b) show the map χ and the corresponding map F_α. $\bar{\mu}^e$ denotes the value of μ^e which maximizes the map F_α, and μ_c^e denotes the value of μ^e where $F_\alpha(\mu_c^e) = 0$. The map F_α in Figure 2 (b) is more hump than the map $\chi(\mu^e)$ in Figure 2 (a). It means that α is greater than 1. It follows from Figure 2 that the maximum of F_α increases as α becomes large while the value of μ_c^e decreases as α becomes large. Therefore there exists α such that the maximum of $F_\alpha(\mu^e)$ is equal to μ_c^e. Then it follows that F_α satisfies the Li-Yorke overshoot condition.

Figure 3

Figure 3 is the bifurcation diagram of μ_t^e where the error correction coefficient α varies smoothly from 0.9 to 1.3 under the specification $\beta = 5.754$, $e_1 = 10$ and $e_2 = 0.95$. Figure 3 shows chaos to occur through a sequence of period doublings as α becomes large.



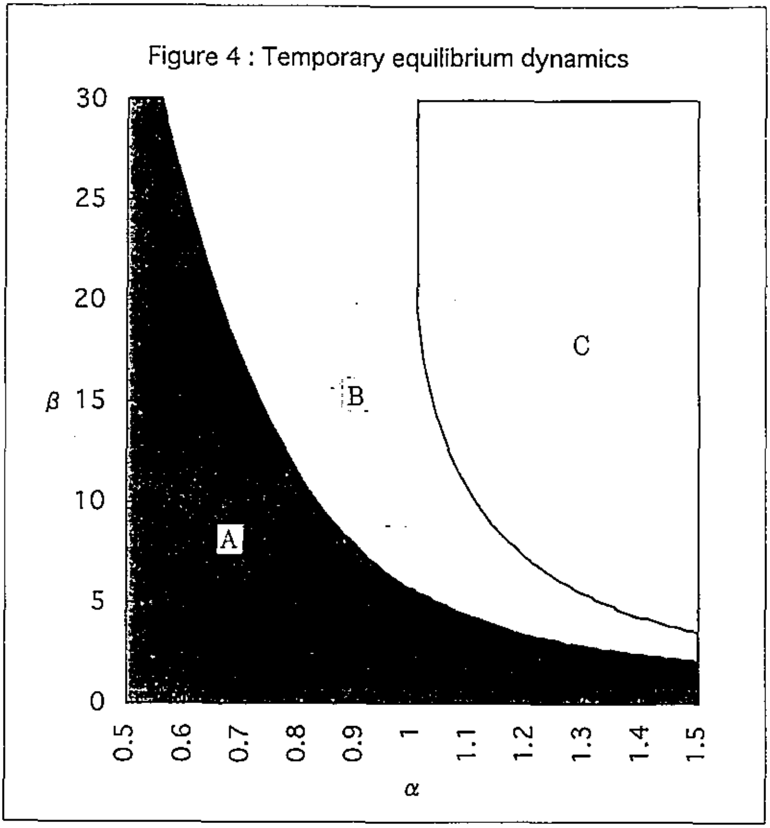


Figure 4 : Temporary equilibrium dynamics

Figure 4

Figure 4 summarizes the propensities of the temporary equilibrium dynamics with adaptive (overreactionary) expectations (4).

A : The temporary equilibrium dynamics (4) converge to the perfect foresight equilibrium μ^*.

B : There are cycles of period 2^n, and chaotic fluctuations.

C : The temporary equilibrium dynamics diverge.

Figure 4 demonstrates that the temporary equilibrium dynamics (4) converge to the perfect foresight equilibrium when β and/or α are small while cyclic and chaotic temporary equilibrium dynamics are caused by any large β and/or any large α.

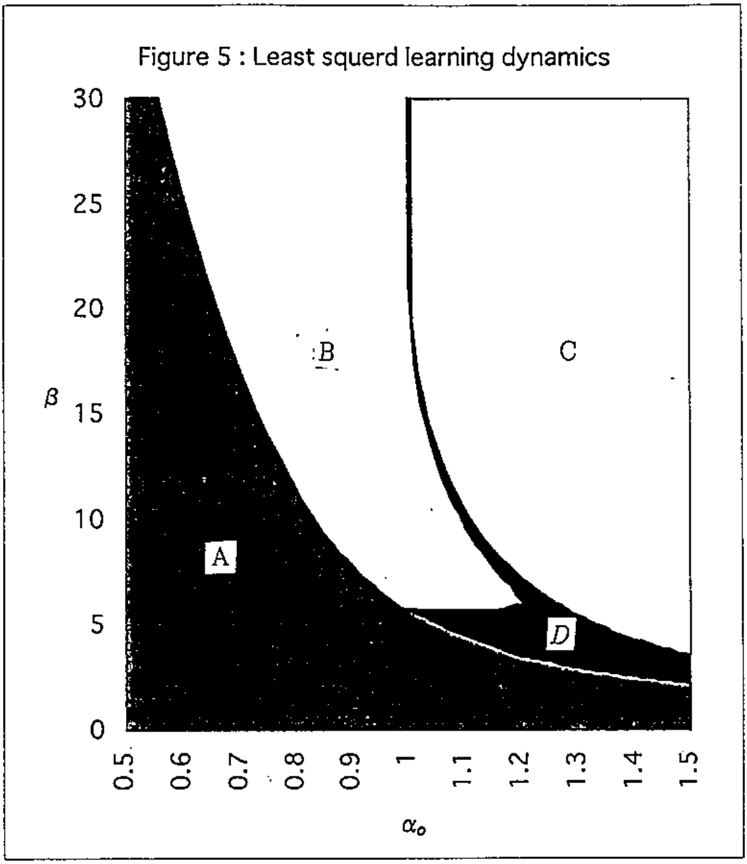


Figure 5

Figure 5 summarizes the propensities of the least squares learning dynamics (8).

A : The least squares learning dynamics converge to the perfect foresight equilibrium μ^*.

B : The least squares learning dynamics occur bounded irregular fluctuations.

C : The least squares learning dynamics diverge.

D : The least squares learning dynamics converge to the perfect foresight equilibrium μ^*.

In Figure 5 α_0 of the horizontal axis is that the initial values of α before agents begin least squares learning. Comparing Figure 4 and Figure 5, we find that in the area D in Figure 5 the least squares learning dynamics converge to the perfect foresight equilibrium μ^* while temporary equilibrium dynamics (4) are unstable.

Figure 6 : Temporary equilibrium dynamics and Least squares learning dynamics

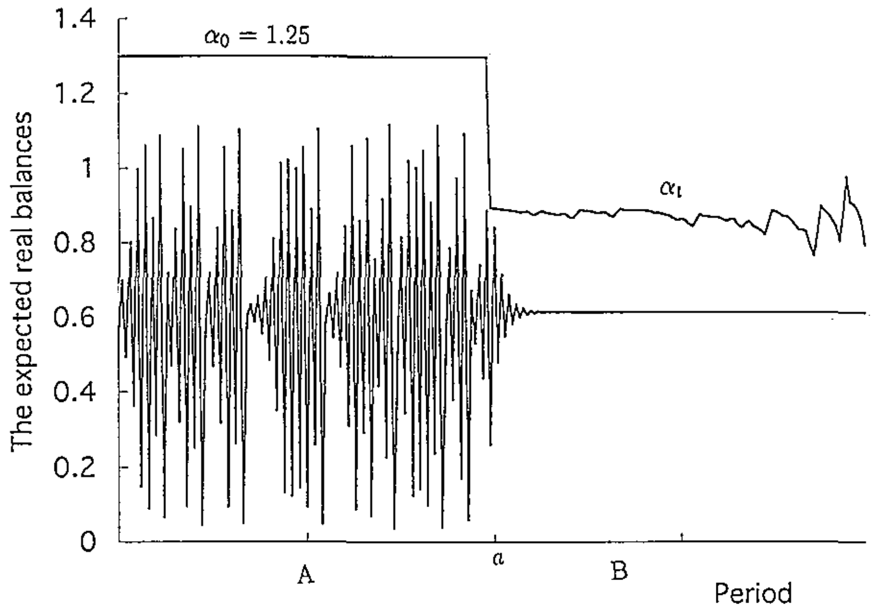


Figure 6

Figure 6 shows both time paths of the temporary equilibrium dynamics and the least squares learning dynamics under a large α_0 and a large β. we assume $\alpha_0 = 1.3$, $\beta = 5$, $e_1 = 10$, and $e_2 = 0.95$. Under this set of parameters, time series of temporary equilibrium dynamics are chaotic (the area A in Figure 6). The point, a is the starting point of the least squares learning. After the agents do the learning of α according to (7), the value of α changes from 1.3 to about 0.88, and so the least squares learning dynamics (8) converge quickly towards the perfect foresight equilibrium μ^* (the area B in Figure 6).

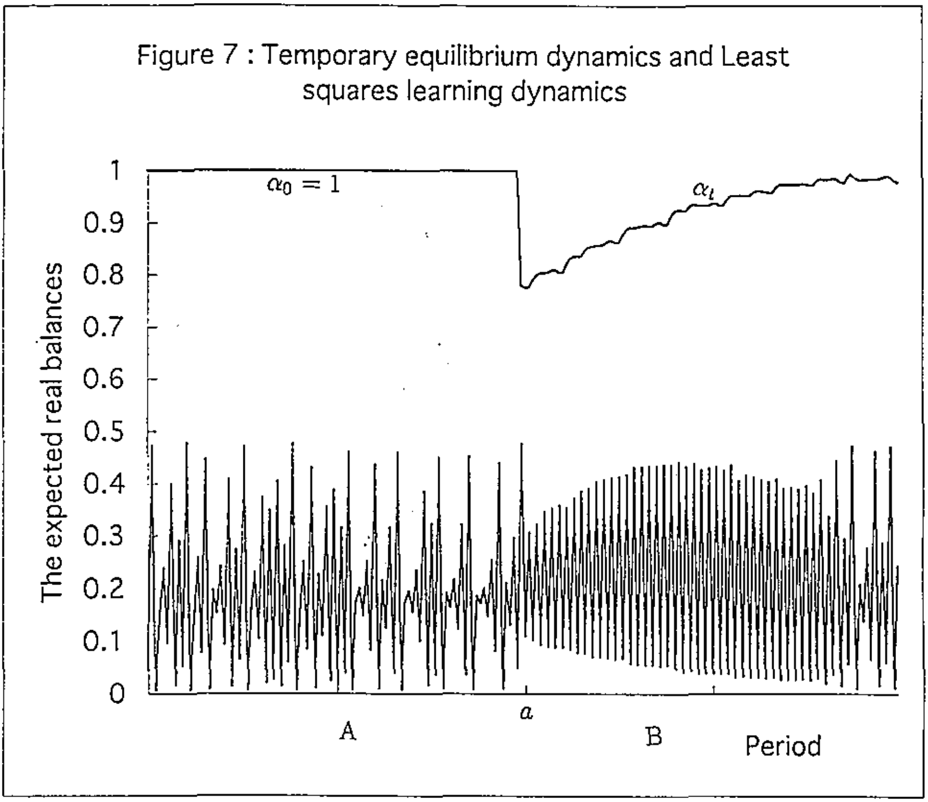


Figure 7

Figure 7 shows both time paths of the temporary equilibrium dynamics and the least squares learning dynamics undera large α_0 and a sufficiently large β. We assume $\alpha_0 = 1$, $\beta = 18.5$, $e_1 = 10$, and $e_2 = 0.95$. Under this set of parameters, time series of temporary equilibrium dynamics are chaotic (the area A in Figure 7). The point, a is the starting point of the least squares learning. After the agents do the learning of α according to (7), the value of α changes from 1 to about 0.77 and the least squares learning dynamics (8) occur bounded irregular fluctuations (the area B in Figure 7). The reason is that the strong wealth effects give cause for endogenous fluctuations.

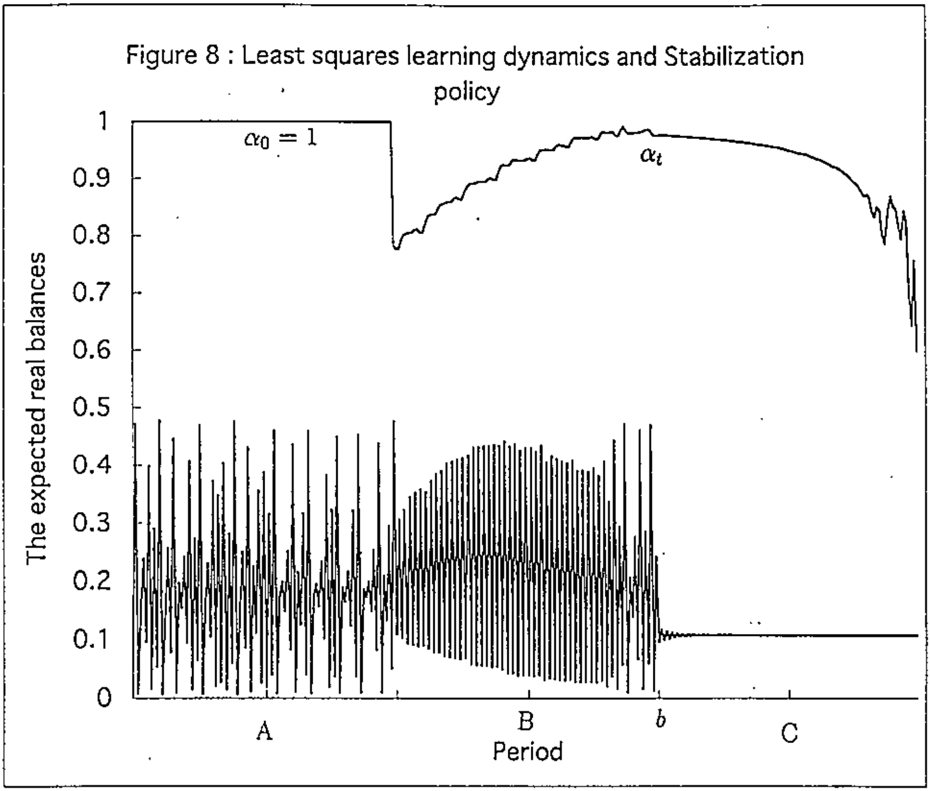


Figure 8

Figure 8 illustrates this stabilizing effect of the monetary policy (an increase of the growth rate of money supply). To analyze the effect of the monetary policy, We set $\beta_2 = 18.5$, $e_1 = 10$, $e_2 = 0.95$, $s = 1$ and $\alpha_0 = 1$. The time series of μ_t^e of the area A in Figure 8 are the temporary equilibrium dynamics, and the time series of μ_t^e of the area B are the least squares learning dynamics before the implementation of the monetary policy under $\beta = 18.5$, $e_1 = 10$, and $e_2 = 0.95$. Both time series of μ_t^e fluctuate irregularly. Then we consider that the government increases the rate of growth of money supply from $s = 1$ to $s > 1$. The point b is the starting point of implementation of the monetary policy. The time series of μ_t^e of the area C are the least squares learning dynamics after the implementation of the monetary policy under $\beta = 18.5$, $e_1 = 10$, $e_2 = 0.95$, and $s = 3.5$. After the implementation of the monetary policy the learning dynamics converge to the new perfect foresight equilibrium.

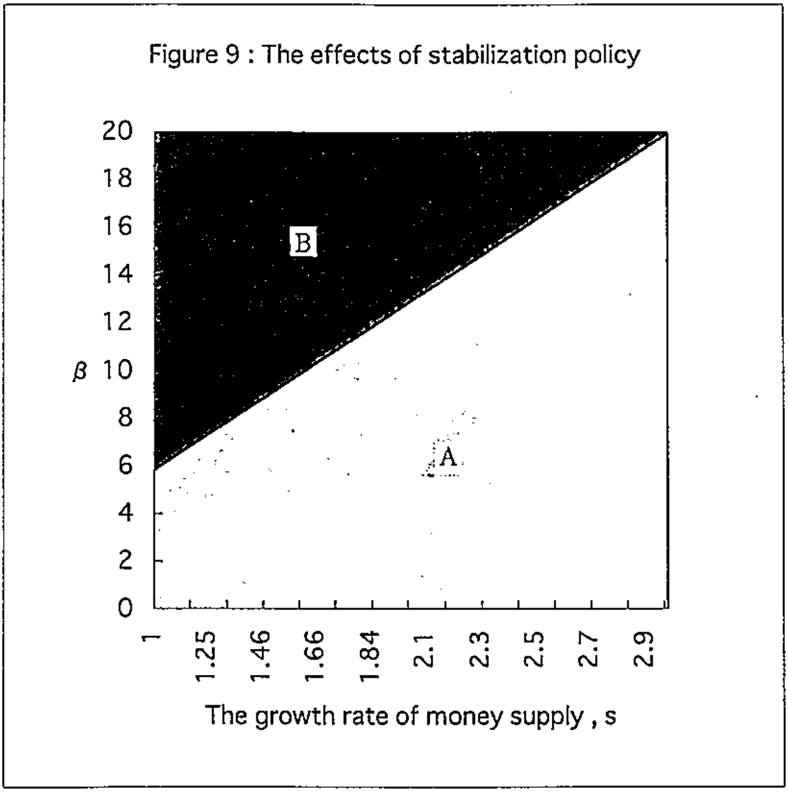


Figure 9

Figure 9 summarizes the stabilizing effects of the monetary policy.

A : The least squares learnig dynamics (13) converge to the perfect foresight equilibrium μ^*.

B : The least squares learning dynamics (13) occur bounded fluctuations.

Figure 9 illustrates that any unstable least squares learning dynamics can be stabilized by the monetary policy.

世代重複モデルにおける期待形成，学習過程および安定化政策

海蔵寺　大成

新古典派経済学の多くのモデルにおいて，いわゆる合理的期待形成が仮定されている。すなわち，経済主体はあらかじめ経済の将来の姿を適切に予想し行動すると考えられている。しかし，現実の世界では経済の行方を的確に予想することは大変困難であり，我々は現実に対する認識をしばしば誤ってしまう。

この論文の目的は，

（1）合理的期待形成を仮定する代わりに，経済主体の予想形成と学習過程を具体的に定式化し，経済主体の予想形成のメカニズムが経済システムを不安定化する可能性があること，

（2）不安定化した経済システムを経済政策によって安定化することが可能であること

を示すことである。

我々は経済主体の予想形成仮説として，適応的予想形成を，学習システムとして最小二乗学習を採用する。最初に，これらの予想形成と学習によって，経済システムの不安定性が増大し，カオスが発生することが示される。次に，金融政策によって，経済システムのカオスが安定化され，経済システムが均衡状態に誘導されることが示される。