『社会科学ジャーナル』 39 〔1998〕
The Journal of Social Science 39 〔1998〕

'Demand Multiplier' versus 'Supply Multiplier' in an Open Economy

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1 Introduction

It is important to recognize that there are two different kinds of multiplier; "demand multiplier" and "supply multiplier." Ordinary multiplier *a la* Keynes and Harrod may be called "demand multiplier" for it explains that a sequential increase in demand or total national income is inspired by the initial creation of effective demand such as investment and exports as bases. And total income is multiplied by $\frac{1}{1-c}$ where c is marginal propensity to consume, since consumption creates production income which in turn induces consumption further. Thus, both bases and multiplier are determined by demand-side factors in an open economy.

In contrast, Professors Akamatsu and Stolper proposed "supply multiplier" for war devastated Japan and Italy respectively. Imported raw materials and other intermediate goods are indispensable inputs for manufacturing production and therefore should be the bases of multiplier formula. To the intermediate goods, labor is added in sequential production stages. The ratio of value added, v, works as a production inspiring power and increases the total outputs by the multiple amount, $\frac{1}{(1-v)}$. There is some difficulty to formalize the "supply multiplier", but its concept is clear.

While the demand multiplier is based upon the demand side theory of open economy, the supply multiplier is backed up by the supply-side theory of open economy. It is interesting to find that each theory recommend different policy measures to rectify imbalance of international payments. Demand multiplier may be suited to a matured and under-consumption economy, whereas the supply multiplier is useful to foster catching-up industrialization of developing economy through the help of foreign direct investment and other international capital inflows.

The demand multiplier *a la* Harrod is briefly presented in Section II, while the origin and concept of supply multiplier is addressed in Section III. Differences of the two multipliers are compared in Section IV. In the Appendix, a model of open economy with

imported intermediate goods is presented diagrammatically, which helps to understand the supply side theory of an open economy.

2 Demand Multiplier

Roy Harrod presented the multiplier in an open economy by combining his "export multiplier" with Keynes' "investment multiplier"[1] as follows:[2] National income, Y, arises (i) from the production of goods sold to consumers at home, (ii) from the production of goods sold abroad, and (iii) from the production of goods which go to swell the stock of capital goods in the country whether fixed or liquid. Let us designate each magnitude of income by C, X, and I respectively.

Next, national income may be disposed of (i) in the purchase of home-made consumable goods, (ii) in the purchase of imported goods which consists only of consumption (here, Kojima assumed that no imported raw materials exist to avoid complication), and (iii) in the form of saving.

The *proportion* of income spent on home-made consumer goods is designated by c (later called as marginal propensity to consume), proportion of income spent on imports by m, and proportion of income saved by s. Since the entire income is disposed of in the afore mentioned ways, $c+m+s \equiv 1$. For the sake of simplicity, government expenditure and tax revenue relations are omitted from our consideration. Thus,

$$Y = C + X + I = (c + m + s)Y \tag{1}$$

Since the amount of income spent on the consumption of home-made goods is equal to the amount of income derived from their production

$$C = cY \tag{2}$$

$$I + X = sY + mY \tag{3}$$

$$\text{or } sY - I = X - mY \tag{4}$$

that is, the excess of exports over imports is equal to the excess of saving over investment. Such relationship between internal and external equilibrium (or disequilibrium) is important to analyze causes of and cures for the imbalance.

[1] J. M. Keynes (1936), p.129.
[2] R. F. Harrod (1939), Chap.6, p.127.

Now, I and X are taken to be multiplicands (or bases). If the amount $(I+X)$ is given by, for example, government public construction expenditure and the increase in foreign demand, effective demand (i.e. national product, national income and employment) is created by that amount as a first round. Then,[3] the person who earns $(I+X)$-income spends $(I+X)c$ to consume, $(I+X)m$ to import and $(I+X)s$ to save. The $(I+X)c$ means secondary increase in effective demand, and is spent as a third round $(I+X)c^2$ amount to consume. If this cumulative increase in effective demand is repeated infinitely, the total increase of national income will be,

$$\begin{aligned} Y &= (I+X)(1+c+c^2+\cdots) \\ \text{or } Y &= (I+X)\frac{1}{1-c} \end{aligned} \tag{5}$$

$\frac{1}{1-c}$ is the multiplier, but properly called the "demand multiplier in an open economy" for it explains the cumulative and sequential increase in effective demand. It should be remembered that the motive power is the initial creation of autonomous effective demand and the key element to create multiple increase of effective demand is the marginal propensity to consume. (Saving and imports are 'leakage' from the process of cumulative income increase.)

3 Supply Multiplier

Late professor Kaname Akamatsu [1896–1974] proposed in 1950 "import multiplier" or "supply multiplier" which differs in character from the "demand multiplier" *a la* Keynes and Harrod mentioned in the previous section.

Immediately after the war, 1945–1950, Japanese economy suffered from serious devastation, especially the shortage of food and other daily necessities and reconstruction materials, which brought about severe inflation and social disorder. There was a considerable number of unemployment consisted of repatriates from abroad while some factories and plants remained to engage in the production of war munitions.

The bottleneck was the lack of raw materials and fuel such as cotton, iron ore and oil which were available solely from abroad. Then Akamatsu expected that the import

[3]See R. F. Harrod (1939), p.119.

of raw materials (intermediate goods in general) is indispensable to get start the operation of factory—the production initiating effect. Moreover, the initial production has propagative and multiplying effect on successive production stages. Akamatsu puts his idea of "import multiplier" as follows:

> Here we must consider first the import effect of the materials for production. Some raw materials, it is assumed, are imported on foreign credit. Labour and other factors of production combined with these materials are employed to produce commodities in the first stage of production. Of the manufactured goods of the first stage, some will be directly consumed as consumption goods, such as cotton yarn produced from raw cotton is consumed as sewing-thread, other yarn will be exported, and the remaining part will be utilized in the next stage of production as intermediary goods, thereby labour and other factors are again employed to produce some more complicated manufactured goods. The second stage of production produces commodities of added value, for example, cotton cloth, of which some part will be consumption goods, some part will be exported and the remaining part will be utilized in the third stage of production, in which this process will be repeated. As these processes continue from the first stage of production to higher stages, the imported raw materials diminish in relative value of the whole product, and at last will be extinguished. Then the process which originated from the importation of raw materials comes to an end, having employed much labour and other factors at each stage of production, which would not have happened unless the original materials had been imported.[4]

Foodstuff is thought to be the intermediate goods to the reproduction of human being. Akamatsu stressed the fact that the supplement of imported foodstuff was important to increase labor efficiency, as follows:

> ··· the lack of efficiency of labour was mostly due to a shortage of foodstuffs; wage earners and salaried men were often compelled to go to farmers in the countryside, instead of to their workshops and offices, to get something

[4]Kaname Akamatsu (1950), p.6.

to eat, or they themselves had to cultivate small patches of land around their houses or anywhere they could find some soil uncultivated. They were both physically and mentally inactive in work and therefore less efficient.

As the ration of foodstuffs increased, and black market prices of food (people could not live on controlled rations) went down, the worker's livelihood became more stable, wages advancing more rapidly than prices. Thus efficiency became gradually higher, as workers could devote themselves to their task without having to search for foodstuffs in exchange for some clothing they really needed for themselves and their family members. When the efficiency of labour increases, production rises, and the greater supply will augment real income and welfare, thus real income and efficiency react on each other successively as cause and effect.[5]

About the same time as Akamatsu did, Professor Wolfang Stolper (University of Michigan at that time) independently proposed an "import multiplier" in 1950 by taking up Italian economy as an object of analysis which experienced similar difficulties as Japan did of underproduction with the shortage of imported raw materials.

Suppose we deal with a country such as Italy, the resources of which other than labor are meagre, but which could produce finished products if it could import sufficient capital in the form both of food stuffs, and of raw materials and semi-finished goods. We can now set up the following highly simplified case, which, nevertheless, seems to depict accurately a large slice of reality.[6]

Let income Y consist, as is usual, of consumption of domestic goods C, investment I. We omit even exports E. But suppose we now consider the *input* relations. In the simplest possible case consumption requires only domestic resources. But suppose investment requires both domestic resources, say labor, and imported resources, say coal, cotton, steel.

In this case we can write:

$$Y = C(d) + I(m_r, d) \qquad (a)$$

where d symbolizes domestic, and m_r foreign resources.

[5] *Ibid.*, p.10.
[6] Wolfgang Stolper (1950), p.290.

Then the increase in income is[7]

$$\Delta Y = \frac{\partial C}{\partial d}\Delta d + \frac{\partial I}{\partial m_r}\Delta m_r + \frac{\partial I}{\partial d}\Delta d \qquad (b)$$

The first item is out of consideration here. If imported intermediate goods is increased (that is, Δm_r) and domestic resources (labor) are added (that is, Δd, in the third item) according as required technological proportion with Δm_r. Thus, imported intermediate goods, Δm_r, bring about multiplied amount of income, ΔY. The relationship between Δm_r and ΔY is something like an *import multiplier*.[8] The argument has depended only on technological input relationships.

The concept of "import multiplier" and/or "supply multiplier" is distinct but it is not easy to formulate them for Akamatsu, Stolper, and others including the present writer as a supply-side theory of an open economy.

The supply multiplier stresses the importance of imported raw materials and other intermediate goods, called Z, as an indispensable inputs for starting manufacturing production. Let v stand for the ratio of value added to total value of a product; therefore, $(1-v)$ is the ratio of intermediate goods to total value of the product.

Now, if the value of Z is available, it makes possible to start the production, adding labor work and produces value added income as large as vZ. Thus the value added ratio, v, is, looking from a different angle, a *production inspiring power.*

The production process continues and produces in the second stage the value added income as large as $vZ \times v = v^2Z$, and in the third stage, v^3Z, and so forth, if v is the same in every production stage. Thus the total income of value added may be

$$Y = \frac{1}{1-v}(vZ) \qquad (6)$$

In addition, it is assumed that domestically produced intermediate goods, the value of it is designated by K, play the same role as imported intermediate goods. Thus,

$$Y = \frac{1}{1-v}(vZ + K) \qquad (7)$$

[7]Stolper, *ibid.*, p.291 note (modified).

[8]Stolper, *ibid.*, p.289: "If imported resources which are technologically necessary in varying degrees for domestic production become available in larger amounts, production will become more efficient, and domestic labor more productive. This will raise domestic real income. In the extreme case, where domestic and foreign resources must be combined in fixed proportions (or within a very limited range of proportions), there can be no rise of real income without increased imports."

It is clear that there is a big difference in character between "demand multiplier" and "supply multiplier." In demand multiplier, the base for the increase of income is investment and exports, i.e., $(I + X)$, and the factor which cumulatively increases income is the people's propensity to consume, or c. In contrast, in supply multiplier, the initiator (or ignition) of production is inputs of intermediate goods imported and domestically produced, i.e., $(vZ+K)$, and the factor which successively inspire following stage production is the value added ratio, or v which should be less than 1. The value added ratio is determined mainly by technological input-output coefficient which varies in different industries and different stage of production process. Since the value added ratio is not constant, it is difficult to formulate such a supply multiplier as $\frac{1}{1-v}$ for the national economy as a whole. Therefore, it may be right not to call "supply multiplier" but simply "input-output relations." But the logic of supply multiplier may be well understandable.

4 Demand Multiplier vs. Supply Multiplier Compared

There is no difference in view of equilibrium conditions of total flow of national income between "demand-side theory" and "supply-side theory" of an open economy. But, different policy measures to remedy disequilibrium are addressed from the two theories.

From the point of view of demand multiplier model, in order to rectify external trade deficits, cuts in domestic effective demand are recommended. If investment I, in equation (1), is reduced, its multiplied amount of total income, that is, $Y = \frac{1}{1-c}I$, is reduced and decreased imports, mY, may improve the trade deficits.

In contrast, the supply multiplier model sees the cause of trade deficits to be the shortage of production and recommends the increase in imports of raw materials and other intermediate goods. This aggravates the trade deficits at first, but later on it increases the volume and reduces cost (international competitiveness) of related production, enabling to increase exports and to rectify trade deficits.

The supply multiplier model is more suited than alternative one to the catching-up industrialization process of developing economy. The worsening of trade deficits in

incipient stage may be overcome by several kinds of assistance from advanced countries; official development aid, export processing zone method, multinational's foreign direct investment-led growth, and so on.

Appendix: A Model of Imported Intermediate Goods

The formal barter model of international trade which includes a traded intermediate good is shown diagramatically as Fig.1.[9]

It is assumed that part of domestic production of commodity 1 is exported and part of domestic consumption of commodity 2 is imported, so that:

$$D_1 = X_1 - E_1 \tag{A1}$$

$$D_2 = X_2 + M_2 \tag{A2}$$

where D_i $(i = 1, 2)$ indicates the domestic consumption of these commodities, X_i the level of output, E_1 exports of good 1 and M_2 imports of good 2.

The production functions for the two commodities are assumed to exhibit constant returns to scale. Commodity X_1, the exportable commodity, is produced with the help of capital and labor, in addition to the imported intermediate good Z_1.[10] Commodity X_2 is produced with capital and labor, but does not use any of the imported intermediate goods.

Hence, the production functions are:

$$X_1 = F_1(L_1, K_1, Z_1) = L_1 f_1(k_1, z_1) \tag{A3}$$

$$X_2 = F_2(L_2, K_2) = L_2 f_2(k_2) \tag{A4}$$

where L_i, K_i denote the labor and capital allocation to the ith sector, the value of Z_1 the allocation of imported intermediate good to sector 1, and k_1 and z_1 represent the capital-labor and import-labor intensities in sector 1. Each unit of X_1 requires Z_1 in fixed proportions as below:

$$\bar{a}_z X_1 = Z_1 \tag{A5}$$

[9] It is a modified version of Hazari, Sgro and Suh (1981), Chap.5. See also Ruffin (1969).

[10] We differ on this point from Hazari *et. al.* who assumes that importable good sector uses imported intermediate good.

Perfect competition prevails in the factor market, resources (labor and capital) are fully employed, and the value of exports equals to the total value of imports.

In Figure 1, in the right hand panel (1), the production possibility frontier TT' is drawn. It is concave to the origin as usual.

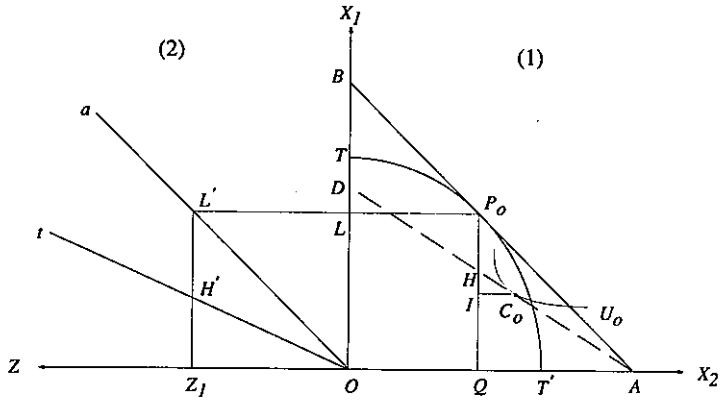

Figure. 1

By totally differentiating equations (A3) and (A4), we obtain:

$$dX_1 = \frac{\partial F_1}{\partial L_1}dL_1 + \frac{\partial F_1}{\partial K_1}dK_1 + \frac{\partial F_1}{\partial Z_1}dZ_1$$
$$dX_2 = \frac{\partial F_2}{\partial L_2}dL_2 + \frac{\partial F_2}{\partial K_2}dK_2$$

By using the above equations and factor endowment conditions, we obtain:

$$-\frac{dX_1}{dX_2} = \frac{p_2}{(1 - \bar{a}_z p_z)} = \frac{p_2}{V} \tag{A6}$$

where $\bar{a}_z$ is an import coefficient and p_i (i = 1, 2 and z) prices, $\bar{a}_z p_z$ represents the cost of imported intermediate goods used in sector 1, and therefore, $(1 - \bar{a}_z p_z) = V$ means the value added price. Further, $-\frac{dX_1}{dX_2}$ represents the slope of the production possibility frontier and is defined as the *domestic* rate of transformation ($DRT = -\frac{dX_1}{dX_2}$). The condition derived above tells us that in competitive equilibrium, the slope of the

transformation curve equals the value added price ratio $\frac{p_2}{V}$, and not $\frac{p_2}{p_1}$ as in the usual model without an imported intermediate good. This is so because producers in sector 1 must take into consideration the price of intermediate good. In other words, producers must undertake production decisions by the value added price which is $V = (1 - \bar{a}_z p_z)$. Thus, the slope of $\frac{p_2}{V}$ represents the budget line facing the producers. In Figure 1, the free-trade production point is denoted by P_o where condition (A6) is satisfied.

Let us compute the quantity of the imported intermediate good required for producing output P_oQ of X_1 in Figure 1. This quantity is derived in the left-hand panel (2) according to the relationship of equation (A5), which is illustrated by the line Oa. Now by drawing the line P_oL', we can determine in panel (2) the amount of intermediate good required for producing X_1. This is indicated by the distance OZ_1.

The imports of OZ_1 must be paid for in terms of exports. In panel (2), the slope of the line Ot represents the price ratio between imported commodity Z and the exportable good X_1. Given the price ratio (as indicated by the slope of Ot), to import OZ_1, exports of the magnitude $H'Z_1$ are required.

The amount of commodity X_1 needed for acquiring the intermediate good must be subtracted from the total production of X_1. Thus $H'Z_1 = P_oH$ is subtracted from the production point P_o. Point H indicates the level of output available for consumption to the economy after the country has paid for the imported intermediate goods.

The consumers equate their marginal rate of substitution $\frac{U_2}{U_1} = \frac{p_2}{p_1} < \frac{p_2}{V}$ (because $V < p_1$). From point A we draw the price ratio $\frac{p_2}{p_1}$ as shown by the slope of the line AD which passes through point H. Consumption equilibrium occurs at C_o and welfare is indicated by U_o indifference curve. The diagram also shows that C_oI of X_2 is imported as final goods in exchange for IH of commodity X_1. Also note that P_oH of commodity X_1 is exported in exchange for OZ_1 of the imported intermediate good.

Now, what gains are brought about by the trade of imported intermediate good ? It is assumed that commodity X_1 cannot be produced without imports of intermediate goods. Therefore, in the absence of trade, the output of X_1 equals zero --- only commodity X_2 can be produced using only domestic capital and labor. The production possibility set reduces to the line OT' in Fig.1, where T' represents the maximum output of X_2 that can be obtained by using the total supply of capital and labor in sector 2. Given our

assumption of full employment, the closed economy production point becomes T' which is also the point of consumption equilibrium. The closed economy welfare is indicated by the social indifference curve U_n which can be drawn through T'. Compared with the closed economy welfare, in the open economy where intermediate goods are traded, welfare is improved to U_o.

The cause of gains from imported intermediate goods is the fact that OZ_1 of needed intermediate goods can be imported as inexpensive as $H'Z_1 = P_oH$ of X_1. Or in other words, Ot, the relative price of imported intermediate good in terms of exportables is lower than the slope of Oa, the technical requirement ratio of intermediate goods in the production of exportables.

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「需要乗数」と「供給乗数」

— 二種の開放経済理論 —

<要約>

小 島 清

ケインズとハロッドが1930年代後半に着想した「投資・輸出乗数」は有名であるが，その性格は「需要乗数」である。投資と輸出（つまり外国の需要）という有効需要が創出されると，その乗数倍の国民所得が増産されることになる。乗数は1/(1-c)であって，限界消費性向cに従って，有効需要＝生産所得が次つぎに波及的に増加することを，フォーミュレートしている。

これに対し，故赤松要教授とストルパー教授が，それぞれ独立に，1950年に，日本とイタリーの戦後の荒廃経済の実情に立脚して，「供給乗数」を提唱した。それは，輸入中間財（綿花，鉄鉱石，石油など）が入手できさえすれば，残存工場が稼働し，次つぎに付加価値（労働）が加えられて，1/(1-v)（ただしv＝限界付加価値率）なる乗数倍の製品(=GNP)を生産できるようになるというのである。

供給乗数のフォーミュレーションにはなお困難が残り，未完成である。供給側アプローチの中心問題たる「中間財貿易モデル」を，補論において，検討しておいた。

需要乗数と供給乗数は，開放経済体系を分析するのに，需要側アプローチと供給側アプローチの二種が存在することを代表するものであり，今後の国際経済学の研究に大きな示唆を与えていることになる。