Another Look at the Golden Turtle

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1. Introduction

This paper empirically examines a spread trading strategy involving the use of gold futures. Spread trades have received little attention in the academic literature despite their widespread use in markets around the world (Poitras, 1997). The spread trade enables the investor to take positions on the relative prices as opposed to the price levels. This has the benefit of being less expensive to execute but most importantly reduces the volatility of the profits generated from the trade.

One such spread trade is the turtle trade (Jones, 1982). A turtle trade involves buying a tailed T-bond spread and selling a position in T-bill futures. This means one takes a long T-bond spread position, and simultaneously go short on a T-bill futures by an amount such that the impact of the absolute price level of T-bonds is immunized. A variation of this trade is to replace the T-bond spread with a gold futures spread and the T-bill futures position with a Eurodollar futures position. This variation is referred to as the golden turtle trade (Poitras, 1987, 1990).

The rationale for the golden turtle trade is outlined in Poitras (1987). In this trade, the implied carry of gold futures will be bounded from above by the Eurocash futures rate and bounded from below by the T-bond spread position. If the gold implied carry exceeds the Eurorate reflected in the Eurocash future, it is possible to fund cash through the Euro market and to invest this cash in the gold futures market by taking advantage of the arbitrage in the implied carry. Thus, it is reasonable to assume that such levels in the gold futures market should not sustain for prolonged periods, consequently, generating a weak bound for the implied carry in gold future prices. If the implied carry of the T-bill rate exceeds the implied carry of the gold futures, Monroe and Cohn (1986) show that it is possible to generate profit from trading based on these implied carries.

We extend the golden turtle spread strategy to a multi-currency framework and find that the trade exhibits statistically significant profits over our sample period of 1996 to 1999.

2. The Trading Strategy

Let us take a position between the Euroyen futures and the Eurodollar futures spread. More specifically, we go long one Euroyen nearby futures contract today (t=0) and close out at t=1 by going short the nearby Euroyen futures contract. Simultaneously, we take a spread position between the nearby Eurodollar futures (short) and a deferred Eurodollar futures contract (long). The Eurodollar spread position is closed out by taking the reverse position at t=1. We will refer to this spread trade strategy as the base case currency spread trade. The trades involved in this base case currency trade strategy are summarized in Table 1.

Table 1: Base Case Currency Spread Trade: Euroyen - Eurodollar Spread

Date	Euroyen Future	Eurodollar Future N	Eurodollar Future T
t=0	Long 1 Euroyen Future contract at $F_{EuroYen}$ (0,N)	Short 1 Eurodollar Future contract at $F_{EuroDollar}$ (0,N)	Long Q_T Eurodollar Future contract at $F_{EuroDollar}$ (0,T)
t=1	Close out position by Selling 1 Euroyen contract at $F_{EuroYen}$ (1,N)	Close out position by Buying 1 Eurodollar contract at $F_{EuroDollar}$ (1,N)	Close out position by Selling Q_T Eurodollar contract at $F_{EuroDollar}$ (1,T)

F(0,N) denotes the price of a nearby futures contract at time *t=0* and *F(1,N)* denotes the nearby futures price at time t=1. The subscript denotes the relevant currency and T refers to the deferred contact.

The profit function for the Euroyen-Eurodollar spread (base case currency spread trade) is given as

$$\Pi(1) = \Delta i - [F(0,N) - F(1,N)]Q_T - [F(1,T) - F(0,T)]$$

$$\Pi(1) = \Delta i - [F(0,N) - F(1,N)]\frac{F(0,T)}{F(0,N)} - [F(1,N)[1 + C(1,N,T)] - F(0,T)]$$

$$= \Delta i + F(1,N)[\frac{F(0,T)}{F(0,N)} - 1] - F(1,N)C(1)$$

$$= \Delta i - \Delta C$$

where F(0,m) is the futures price at the time 0 for delivery at time m (maturity), F(0,N) is the futures price for the nearby delivery (m=N) at t=0, F(0,T) is the futures price for the deferred (m=T) delivery at t=0, Δi denotes interest reflected in the Euroyen futures position and Π denotes profit. The implied carry at t=0, C(0), is defined as $C(0) = \{\frac{F(0,T) - F(0,N)}{F(0,N)}\} \times \frac{365}{tsm}$ where *tsm* is the time from delivery on the nearby contract to the delivery on the deferred contract. For simplicity, we have set, $Q_T = F(0,T) / F(0,N)$ and $\Delta C = C(1) - C(0)$. Notice the profits are not a direct function of the price level but of the interest rate (yen) and the change in the implied carry from the Eurodollar futures spread.

This extends to a trading strategy involving multiple-currency "golden turtles". Here, we will only focus on the two-currency case: the Japanese Yen and the US dollar to illustrate how the trade works. The Euroyen-gold spread leg is summarized in Table 2. It simply involves replacing the Eurodollar spread in the base case currency trade with a gold futures spread.

Table 2: Euroyen-Gold Spread Leg

Date	Euroyen Future	Gold Future N	Gold Future T
t=0	Long 1 Euroyen Future contract at $F_{EuroYen}$ (0,N)	Short Gold Future contract at F_{Gold} (0,N)	Long Q^{Gold}_T Gold Future contract at F_{Gold} (0,T)
t=1	Close out position by Selling 1 Euroyen contract at $F_{EuroYen}$ (1,N)	Close out position by Buying Gold contract at F_{Gold} (1,N)	Close out position by Selling Q^{Gold}_T Gold contract at F_{Gold} (1,T)

The profit function for the Euroyen-gold spread leg, Π_{Yen}, is

$$\Pi_{Yen} = \Delta i - [F_{Gold}(0,N) - F_{Gold}(1,N)]Q_T - [F_{Gold}(1,T) - F_{Gold}(0,T)]$$

$$\Pi_{Yen} = \Delta i - [F_{Gold}(0,N) - F_{Gold}(1,N)]\frac{F_{Gold}(0,T)}{F_{Gold}(0,N)} - [F_{Gold}(1,N)[1 + C_{Gold}(1,N,T)] - F_{Gold}(0,T)]$$

$$= \Delta i + F_{Gold}(1,N)[\frac{F_{Gold}(0,T)}{F_{Gold}(0,N)} - 1] - F_{Gold}(1,N)C_{Gold}(1)$$

$$= \Delta i - \Delta C_{Gold}$$

Again for simplicity, we have set, $Q^{Gold}{}_T = F_{Gold}(0,T) / F_{Gold}(0,N)$ and $\Delta C_{Gold} = C_{Gold}(1) - C_{Gold}(0)$ so that the profit is not a function of the price level.

The Eurodollar spread-gold spread leg is summarized in Table 3. Here, we simply replace the Euroyen position in the base case currency trade with a gold futures spread.

Table3: Eurodollar Spread-Gold Spread Leg

Date	Gold Future N	Gold Future T	Eurodollar Future N	Eurodollar Future T
t=0	Short $Q^{Gold}{}_T$ Gold Future contract at $F_{Gold}(0,N)$	Long Gold Future contract at F_{Gold} (0,T)	Short $Q^{Dollar}{}_T$ Eurodollar Future contract at $F_{EuroDollar}$ (0,N)	Long Eurodollar Future contract at $F_{EuroDollar}$ (0,T)
t=1	Close out position by Buying $Q^{Gold}{}_T$ Gold contract at F_{Gold} (1,N)	Close out position by Selling Gold contract at F_{Gold} (1,T)	Close out position by Buying $Q^{Dollar}{}_T$ Eurodollar contract at $F_{EuroDollar}$ (1,N)	Close out position by Selling Eurodollar contract at $F_{EuroDollar}$ (1,T)

The profit function for the Eurodollar spread-gold spread, Π_{Dollar}, becomes the following.

$$\Pi_{Dollar} = -[F_{Gold}(0,N) - F_{Gold}(1,N)]Q^{Gold}{}_T - [F_{Gold}(1,T) - F_{Gold}(0,T)]$$

$$-[F_{Dollar}(0,N) - F_{Dollar}(1,N)]Q^{Dollar}{}_T - [F_{Dollar}(1,T) - F_{Dollar}(0,T)]$$

$$\Pi_{Dollar} = -[F_{Gold}(0,N) - F_{Gold}(1,N)]\frac{F_{Gold}(0,T)}{F_{Gold}(0,N)} - [F_{Gold}(1,N)[1 + C_{Gold}(1,N,T)] - F_{Gold}(0,T)]$$

$$-[F_{Dollar}(0,N) - F_{Dollar}(1,N)]\frac{F_{Dollar}(0,T)}{F_{Dollar}(0,N)} - [F_{Dollar}(1,N)[1 + C_{Dollar}(1,N,T)] - F_{Dollar}(0,T)]]$$

$$= F_{Gold}(1,N)[\frac{F_{Gold}(0,T)}{F_{Gold}(0,N)} - 1] - F_{Gold}(1,N)C_{Gold}(1)$$

$$-[F_{Dollar}(1,N)[\frac{F_{Dollar}(0,T)}{F_{Dollar}(0,N)} - 1] - F_{Dollar}(1,N)C_{Dollar}(1)]$$

$$= \Delta C_{Gold} - \Delta C_{Dollar}$$

where, $Q^{Gold}{}_T = F_{Gold}(0,T) / F_{Gold}(0,N)$, $Q^{Dollar}{}_T = F_{Dollar}(0,T) / F_{Dollar}(0,N)$,
and $\Delta C_{Gold} = C_{Gold}(1) - C_{Gold}(0)$.

The profit function for the two combined legs, $\Pi_{Yen\text{-}Dollar}$, involves a hedge ratio between the Yen and Dollar positions. Let the hedge ratio be denoted as H. Then the combined profit function of the Euroyen-gold spread and Eurodollar spread-gold spread positions is given as

$$\Pi_{Yen-Dollar} = (\Delta i_{Yen} - \Delta C_{Gold}) + H(\Delta C_{Dollar} - \Delta C_{Gold})$$

Notice that if the hedge ratio is set equal to one, the above profit function reduces to the base case currency trade profit function.

3. Bounds and Triggers

We assume from historical pricing data that the gold implied carry is weakly bounded below by the implied carry in the Euroyen futures spread and bounded from above by the Euroyen interest rate. The same is assumed to hold for the Eurodollar futures spread and Eurodollar interest rate.

$$C_{Yen} \leq_w C_{Gold} \leq_w \Delta i_{Yen}$$

$$C_{Dollar} \leq_w C_{Gold} \leq_w \Delta i_{Dollar}$$

From the historical relationship above we have the following bounds $C_{Dollar} \leq_w C_{Gold} \leq_w \Delta i_{Yen}$.

Now we propose a triggering strategy as in Poitras (1987) to exploit the pricing inefficiency generated by the convenience yield of gold futures. The trade consists of a spread trade between the gold-Eurodollar spread trade and the Euroyen-gold futures spread trade. We only enter a trade if the base case currency spread, $\Delta i - \Delta C$, is larger than 80 basis points. If the gold implied carry approaches either of the bounds within a 20% margin, we enter into an appropriate trade. In other words, if the gold futures implied carry approaches the upper bound (Euroyen interest rate), then go short the Euroyen and long the Eurodollar spread. At the same time, we take a long position in the Eurodollar spread and short on the gold spread. As the gold futures spread reverts back away from the bound, the trades are reversed to lock in a profit. On the other hand, if the gold futures implied carry approaches the lower bound, we take opposite positions from the upper bound trade and again reverse the positions as the gold carry reverts back away from the weak boundaries. Note we close out of the trade when the implied carry of the gold futures reverts back within the bounds out of the 20% range.

4. Data and Estimation Results

We used daily TIFFE closing prices for the Euroyen and Eurodollar futures. Daily gold futures data was obtained from the TOCOM. The contracts used for the triggering strategy were from 1996.8.30-1999.12.13 (1056 observations). The implied carry was calculated from the nearest two contracts available and are rolled over when the nearby contract expires. The nearest Euroyen futures contract was used to calculate the cash rate, and similarly rolled over when the contract expires.

We provide two different estimates for the hedge ratio, H. The unconditional hedge ratio was obtained by calculating the covariance and variance over 30 days. The 30 day window is shifted on a daily basis. The GARCH based conditional hedge ratio (Kroner and Sultan, 1993) is estimated by using 30 days of data. The 30 day window is shifted by 30 days. The hedge ratio is defined as the following. For one Euroyen spread-gold spread contract, we purchase H number of Eurodollar- gold spread contracts.

The minimum variance version of the hedge ratio, H_{mv}, is given as

$$H_{mv} = \frac{Cov(\Delta i_{EuroYen} - \Delta C_{Gold}(1), \Delta C_{EuroDollar}(1) - \Delta C_{Gold}(1))}{Var(\Delta C_{EuroDollar}(1) - \Delta C_{Gold}(1))}$$

The minimum variance hedge ratio can be derived by minimizing the combined profit function, $\Pi_{Yen-Dollar} = H\Pi_{Dollar} + \Pi_{Yen}$.

Minimizing the variance of the portfolio by selecting the appropriate hedge ratio, H_{mv}, gives us the following assuming the second order conditions are satisfied.

$$\frac{\partial Var(\Pi)}{\partial H}$$

$$= \frac{\partial Var(E[\Pi^2] - E[\Pi]^2)}{\partial H}$$

$$= 2Cov(\Pi_{Dollar}, \Pi_{Yen}) + 2HVar(\Pi_{Dollar})$$

Setting this equal to zero and solving results in the hedge ratio,

$$H_{mv} = \frac{Cov(\Pi_{Dollar}, \Pi_{Yen})}{Var(\Pi_{Dollar})}$$

The conditional version of the hedge ratio is estimated via a bivariate GARCH model. The mean equations of the GARCH model are given as

$$\Pi_{Yen,t} = \alpha_{0Yen} + \varepsilon_{Yen,t}$$

$$\Pi_{Dollar,t} = \alpha_{0Dollar} + \varepsilon_{Dollar,t}$$

where, Π_{Yen} and Π_{Dollar} denote the Euroyen-gold spread and Eurodollar spread-gold spread profit positions respectively. ε are the error terms which are distributed normally.

$\begin{bmatrix} \varepsilon_{Yen,t} \\ \varepsilon_{Dollar,t} \end{bmatrix} | \Psi_{t-1} \sim N(0, \Omega_t)$ where Ψ_{t-1} is the information set at time t-1. The conditional variance-covariance matrix is given as

$$\Omega_t = \begin{bmatrix} h^2{}_{Yen,t} & h_{Yen-Dollar,t} \\ h_{Yen-Dollar,t} & h^2{}_{Dollar,t} \end{bmatrix} = \begin{bmatrix} h_{Yen,t} & 0 \\ 0 & h_{Dollar,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{Yen,t} & 0 \\ 0 & h_{Dollar,t} \end{bmatrix}$$

$$h^2_{Yen,t} = c_{Yen} + a_{Yen}\varepsilon^2_{Yen,t-1} + b_{Yen}h^2_{Yen,t-1}$$

$$h^2_{Dollar,t} = c_{Dollar} + a_{Dollar}\varepsilon^2_{Dollar,t-1} + b_{Dollar}h^2_{Dollar,t-1}$$

h^2 denotes the conditional variance (covariance). c, a, b, α, ρ, are parameters to be estimated. The hedge ratio becomes $H_{GARCH,t} = \frac{h_{Yen-Dollar,t}}{h^2_{Dollar,t}}$.

Table 4: Hedge Ratios

	GARCH H	Min Var H
Average	0.8462	0.9335
Standard Dev.	0.5418	0.0785
Maximum	2.8800	0.6194
Minimum	0.1292	0.0638

Average of GARCH Variance-Covariance Parameter Estimates

	Coefficient on h_{Yen}	Coefficient on h_{Dollar}	Coefficient on $h_{Yen\text{-}Dollar}$
Average	-0.005	-0.0047	-0.0087
Standard Dev.	0.0813	0.0813	0.081

The profits are all statistically significant at standard levels of confidence.

Table 5: Profit and Loss

	GARCH P/L	Min Var P/L	No Hedge P/L
Mean	0.450909	0.03394	0.01552
Standard Dev.	0.069549	0.053075	0.001

Profit/Loss is given as $\Pi_{Yen-Dollar} = \Delta i_{Yen} - \Delta C_{Gold} + H(\Delta C_{Dollar} - \Delta C_{Gold})$ and calculated on a daily basis. GARCH P/L uses $H = \hat{h}_{Yen-Dollar,t} / \hat{h}^2_{Dollar,t}$, Min Var P/L uses $H = \frac{Cov(\Pi_{Dollar}, \Pi_{Yen})}{Var(\Pi_{Dollar})}$, and No Hedge P/L uses $H=1$.

Note, the profits generated from the GARCH hedge ratio based strategy provides us with the largest profits on average. Thus we provide some evidence that time-varying conditional hedge ratios outperform the minimum variance unconditional hedge ratio as often discussed in the literature (Kroner and Sultan, 1993).

Acknowledgements

The authors would like to thank the participants at the September 2000 Colloquium of the Japan Association of International Economics for comments on an earlier version of the paper.

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ゴールデン タートルに関する一考察

〈要　約〉

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本研究は、Poitras によって提唱された金先物とユーロ先物を利用したスプレッドトレード（ゴールデンタートル）を複数の通貨に対して行うことで、利益を統計的に優位に得られることを実証研究を通して検証する。スプレッドトレードは、先物の絶対価格ではなく、先物価格の相対的変化に注目するため、トレードに必要な資金及びトレードによる利益のボラティリティーが少ないという特徴をもつ。こうした利点を持つために実務家の間で広く使われているが、この分野についての研究はほとんど見られない。ゴールデンタートルで使われるスプレッドは、金先物の絶対的価格の損益変化を相殺する形を取り、テーリングされたスプレッドと呼ばれ、また金先物のキャリーを表している。このキャリーはユーロ先物のレートとテーリングされた国債のスプレッドの間を変化するという歴史的事実が存在する。Poitras はこれらのキャリーの性質を利用した裁定取引の可能性について述べた。本研究は、この取引をさらに拡張して、米ドルと金先物を使ったゴールデンタートルの他に、日本と金先物を利用した二つのゴールデンタートルを行った場合についての実証研究を行う。