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Warrants as Synthetic Stock Options: A Note*

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1. Introduction

Do stock options provide an incentive for management to create value and improve corporate performance? The question remains unanswered. Campbell and Wasley (1999) argue that stock option type incentive contracts were not particularly useful in the case of Ralston Purina. Although Ralston Purina required roughly a 60% increase in stock prices over ten years for the stock options to be exercised, this increase in stock prices implied a required return on equity of about 7.7% which is lower than its cost of capital. In other words, little is required of management to fulfill the target stock price requirement imbedded in the executive stock option. On the other hand, Dial and Murphy (1995) find executive equity based compensation created value for the shareholders of General Dynamics. In Japan, stock options were legalized in June 1997. The history is still too short to tell whether stock options will be an important part of Japanese corporate culture. This paper examines whether synthetic stock options as opposed to stock options are of value. In particular we focus on whether such market based incentives are cost effective using the framework developed in Meulbroek (2001).

A large number of Japanese corporates now engage in the use of stock options. And an important practical issue revolving around stock options is how they should be valued and then recorded for accounting purposes. FASB (US accounting standards) recommends the use of the Black-Scholes (BS) model. This has come under attack from many quarters. The BS model does not take into account early exercise as it is a closed-form solution for pricing European options. Thus, at first pass, we would think that the

BS model would ***undervalue*** stock options and thus be recorded as an expense below its true value. But stock options involve other characteristics which are not modeled in the BS framework. For example, the stock option is not transferable and not tradable. In other words, if the executive leaves the company, he or she cannot take the stock option with them making the option less valuable. On a related note, the option cannot be hedged. Also in times where there are shocks to personal wealth, the executive will most likely exercise the option early. All of these characteristics point towards early exercise and thus lead us to believe the BS model will ***overvalue*** the stock option. Carpenter (1998) empirically investigates the issue of early exercise by comparing a utility based valuation model incorporating risk aversion parameters to a modified binomial tree model to explain actual exercise of stock options for a sample of 40 corporate stock option plans in the U.S. The utility based model did not outperform the binomial tree model.

Before stock options were legalized in Japan, companies experimented with instruments referred to as synthetic stock options. As Sony was pioneering in the use of this instrument, it is often called a Sony type synthetic stock option. The synthetic stock option is an interesting case of financial innovation and engineering in Japan. A corporate such as Sony would issue a corporate bond tied to a warrant as a "sweetener." A security house, in this case, Yamaichi Securities, would purchase the bond and warrant. As the warrant is detachable, the bond is then sold to investors and the warrant sold back to Sony. This warrant is then distributed to the executives. This paper focuses on the cost efficiency of these warrants or synthetic stock options. For our numerical case study, we choose to look at the synthetic stock options issued by Sony in the mid-1990's.

2. Warrant Valuation

Meulbroek (2001) takes the value of the stock option calculated using the Black-

Scholes model (FASB benchmark) and compares it to a representative executive's valuation of the stock option. If the executive's wealth position is tied into the value of the firm, then their portfolio is less diversified. An executive receiving a large fraction of their compensation of equity or stock options would thus find their fortunes tied to the market price of their company's stock. From the executives perspective, this undiversified wealth position is not always desirable. As a consequence the executive would place less value on stock options. However, it is important to note that in order for the stock option to play an effective role as a market based incentive, the executive's wealth should be closely aligned with the value of the firm.

In equilibrium, risk adjusted excess returns on all assets are equal. It follows that the risk adjusted returns on the market and on the shares of stock held by an undiversified executive should be the same.

$$\frac{r_m - r_f}{\sigma_m} = \frac{r^U_{Sony} - r_f}{\sigma_S}$$

where, r_m is the return on the market portfolio, r^U_{Sony} is the return on Sony as required by an undiversified executive, and σ_i for $i=m$, S is the volatility for the market return *(m)* and Sony stock return *(S)*. Rearranging and subtracting the market return on Sony stock, r_{Sony}, gives us

(1) $$r^U_{Sony} - r_{Sony} = \left[\frac{\sigma_S}{\sigma_m} - \beta_{Sony}\right](r_m - r_f).$$

The return on the Sony warrant, R_{Sony}, can be written as,

(2) $$R_{Sony} = \frac{\frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 W}{\partial S^2} + r_{Sony} S \frac{\partial W}{\partial S} - \frac{\partial W}{\partial \tau}}{W}$$

where W is the value of the warrant and τ is the time to maturity. Note this is simply the fundamental valuation equation (partial differential equation) for the Black-Scholes model.

We know that the volatility for the Sony warrant can be expressed as $\sigma_{SW} = \dfrac{\sigma_S S \dfrac{\partial W}{\partial S}}{W}$. Hence, the risk adjusted return on the warrant can be formulated as

$$\text{(3)} \qquad \frac{R_{Sony} - r_f}{\sigma_{SW}} = \frac{\frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 W}{\partial S^2} + r_{Sony} S \frac{\partial W}{\partial S} - \frac{\partial W}{\partial \tau} - r_f W}{\sigma_S S \frac{\partial W}{\partial S}}.$$

As before, the risk adjusted warrant return should equal the risk adjusted market return. After rearranging this gives us,

$$\text{(4)} \qquad \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 W}{\partial S^2} + [r_{Sony} - \frac{\sigma_S}{\sigma_m}(r_m - r_f)]S \frac{\partial W}{\partial S} - \frac{\partial W}{\partial \tau} - r_f W = 0.$$

Assuming the CAPM holds and substituting equation (1) into (4), we have,

$$\text{(5)} \qquad \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 W}{\partial S^2} + [r_f - (r_{Sony}^U - r_{Sony})]S \frac{\partial W}{\partial S} - \frac{\partial W}{\partial \tau} - r_f W = 0.$$

Notice, this is equivalent in form to the fundamental valuation equation for a call option on a (proportional) dividend paying stock. Instead of the Black-Scholes model, we use the Black-Scholes based warrant pricing model [Lauterbach and Schultz (1990)].

$$W = \left(\frac{N}{N/\gamma + M}\right)\left[(S\exp(-(r_{Sony}^U - r_{Sony})\tau) + \frac{M}{N}W)N(d_1) - \exp(-r_f\tau)XN(d_2)\right]$$

$$d_1 = \frac{\ln\left([S + \frac{M}{N}W] \Big/ X\right) + [r_f - (r_{Sony}^U - r_{Sony})]\tau}{\sigma_S\sqrt{\tau}} + \frac{\sigma_S\sqrt{\tau}}{2}$$

$$d_2 = d_1 - \sigma_S\sqrt{\tau}$$

where N is the number of shares outstanding of the underlying stock, M is the number of warrants issued, γ is the number shares that can be purchased with each warrant, r_f is the riskfree rate, and τ is the time to maturity. W is the warrant price, S is the underlying stock price, X is the strike price. $N(.)$ is the cumulative normal distribution function. Dividends are assumed to be zero for the purpose of this paper. This will affect the absolute pricing of the warrants, however, we are only concerned with relative pricing in this paper. Although, relatively sophisticated models exist to price warrants and stock options, practitioners are more likely to use simpler closed-form models for accounting purposes. We assume the Black-Scholes model is a good first order approximation. Given the warrant prices from the above equation we can evaluate the cost efficiency of the warrant-type stock option as

$$\frac{W(S^U,\tau,\sigma_{Sony},r_f,X)}{W(S,\tau,\sigma_{Sony},r_f,X)}.$$

We will refer to this ratio as the cost efficiency ratio.

In order to obtain, $W(S^U,\tau,\sigma_{Sony},r_f,X)$, the warrant price for the undiversified executive, we assume that $S^U = S\exp(-(r^U_{Sony} - r_{Sony})\tau)$ and make appropriate substitutions in the above warrant pricing model. To implement the above warrant pricing model we require an estimate of the equity volatility (stock return volatility), σ_S. This is done by using three different methods. The historical volatility, $\sigma_{S,HIST}$, is the sample standard deviation of daily returns over a time span of one year.

$$\sigma_{S,HIST} = \sqrt{252}\sqrt{\frac{\sum_t (r_{Sony,t} - \bar{r})^2}{K}}$$

where $r_{Sony,t}$ is the daily return for day t, $\bar{r}$ is mean return over the sample (one year), and K is the number of usable observations. We assumed a 252 trading day year to obtain an annual volatility estimate.

The second approach was to use a GARCH(1,1) model [Bollerslev (1986)] to estimate an in-sample standard deviation. Due to space limitations we will abstract from any discussion of the GARCH(1,1) as it is a standard time series model. Summary of GARCH(1,1) parameter estimates are available from the author upon request. Our third approach involves estimating an EGARCH(1,1) model [Nelson (1991)] and obtaining an average in-sample volatility estimate. The EGARCH(1,1) model estimated in this paper is

$$r_{Sony,t} = \alpha_0 + \alpha_1 r_{Sony,t-1} + \sigma_t \varepsilon_t$$
$$\ln \sigma_t^2 = \beta_0 + \beta_1 \ln \sigma_{t-1}^2 + \beta_3 (| \varepsilon_{t-1} | - \sqrt{2/\pi} - \beta_2 \varepsilon_{t-1})$$

where ε_t~N(0,1). The mean equation is assumed to take an AR(1) process. β_1 reflects the persistence in the volatility. If $-\beta_3\beta_2$ <0, then a positive shock, ε_t >0, would result in a decrease in volatility. Thus, $-\beta_3\beta_2$ introduces an asymmetric effect of a shock on volatility. Although we do not explicitly assume the volatility is governed by a stochastic process [Duan (1995), and Marsh and Kuwahara (1992)], the warrant is at-the-money at the time of issue and thus the Black-Scholes model will hold as an approximation [Takezawa and Shiraishi (1996)].

3 . Data and Empirical Results

Table 1: Summary Statistics

	Mean	Std. Dev	Min	Max	Beta
Sony	-0.0459	1.9263	-5.9189	9.4003	
TOPIX	-0.0371	1.1330	-4.8233	4.9068	0.965
Nikkei	-0.0355	1.3776	-5.7627	6.0775	0.782

Daily returns are calculated as log relative prices. Sample is from October 1994 to September 1995. Mean is the mean daily return in percent. Std Dev is the standard deviation of the daily returns not adjusted to an annual rate. This is the daily historical volatility estimate. Min and Max are the minimum and maximum returns over the sample period. Beta is estimated by regressing the Sony stock return on each index return.

The first synthetic stock option was issued in the Fall of 1995 to 36 Directors in Sony. Each warrant entitled the holder to purchase 2 million yen worth of Sony's common stocks at a strike of 5,330 yen. The life of the warrant spanned from October 1, 1995 to August 31, 1999. Daily returns for Sony stock and the TOPIX index are calculated as daily log relative closing prices from October 1994 to September 1995. The beta for Sony is estimated by running a regression of the daily Sony stock returns on each index return over the entire sample period. The beta against the TOPIX index is estimated at 0.96 and 0.78 against the Nikkei index (Table 1). As a riskfree rate we used 3% which is approximately the JGB yield during this time frame. The market premium is assumed to be 5%.

The historical volatility estimate (not annualized) is found in Table 1 along with the estimate of the beta. The GARCH and EGARCH models are fit to the daily return series for the same one year sample period. The EGARCH parameter estimates are given Table 2.

Table 2: EGARCH Conditional Variance Parameter Estimates

	Sony	TOPIX	Nikkei
β_0	-0.0019 (4.51)	-0.3052 (2.06)	-0.6800 (2.07)
β_1	0.9411 (0.32)	0.9634 (54.68)	0.9228 (25.72)
β_2	-0.2286 (1.29)	0.7552 (1.82)	0.2142 (2.06)
β_3	0.4887 (4.89)	0.1435 (1.81)	0.3257 (4.88)
LB	6.59	4.27	0.09
LBSQ	1.61	3.65	7.85
SKEW	-0.09	-0.09	0.41
KURT	5.41	7.46	5.69

LB is the Ljung-Box statistic for the standardized residuals at lag one. LBSQ is the Ljung-Box statistic for the squared standardized residuals at lag one. SKEW and KURT are the skewness and kurtosis statistics of the standardized residuals. t-statistics are in parentheses.

The empirical evidence suggests that Sony executives did not value the warrants at a discount even when assuming they are completely undiversified. Using the historical volatility estimates our efficiency ratios were 98% and 99% for the TOPIX and Nikkei index respectively. Similar results can be seen for the efficiency ratios calculated based on the GARCH and EGARCH volatility estimates. This is due to the relatively narrow spread in required returns for Sony stock by the executive and the market (investors).

Table 3: Efficiency Ratio

$$\text{Efficiency Ratio} = 100\text{x}\frac{W(S^{U},\tau,\sigma_{Sony},r_f,X)}{W(S,\tau,\sigma_{Sony},r_f,X)}$$

	Historical Volatility	GARCH Volatility	EGARCH Volatility
TOPIX	98.6%	98.4%	99.5%
Nikkei	99.0%	98.8%	97.6%

The efficiency ratio is calculated using different estimates of volatility for two different proxies for the market (stock index). Historical volatility refers to use of the historical standard deviation for Sony stock returns and the index (market) return. EGARCH refers to use of the in-sample volatility estimate for Sony stock returns and the index (market) return. Warrant prices and efficiency ratios are then obtained under each of the different volatility scenarios.

Notes

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疑似ストックオプションに関する一考察

〈要　約〉

竹澤　伸哉

近年、ストックオプションはコスト効率面において報酬制度として有効かどうか議論されている。ソニーの疑似ストックオプションをケースとして取り上げ、このコストの有効性の実証分析を行う。