

Open Lecture

## **The Multi-Fractal Model of Asset Returns: GMM Estimation and Volatility Forecasting**

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Multi-fractal processes have been proposed as a new formalism for modeling the time series of returns in finance. The major attraction of these processes is their ability to generate various degrees of long memory in different powers of returns - a feature that has been found to characterize virtually all financial prices. Furthermore, elementary variants of multi-fractal models are very parsimonious formalizations as they are essentially one-parameter families of stochastic processes. The aim of this lecture is to provide the characteristics of a causal multi-fractal model (replacing the earlier combinatorial approaches discussed in the literature), to estimate the parameters of this model and to use these estimates in forecasting financial volatility. We use the auto-covariances of log increments of the multi-fractal process in order to estimate its parameters consistently via GMM (Generalized Method of Moment). Simulations show that this approach leads to essentially unbiased estimates, which also have smaller root mean squared error than those obtained from alternative methods. Using these estimates in out-of-sample forecasting of volatility for a number of important financial assets, we also compare the performance of the multi-fractal model with forecasts derived from GARCH and FIGARCH models as well as historical volatility. Overall, the multi-fractal forecasts come out very favourably from this competition: they dominate all other forecasts in two out of four cases considered, while in the remaining two cases all four methods have about the same success.