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A Simple Dynamic Solution to a Politician's Dilemma between Unemployment and Environment

SungSup Ra

Abstract

This paper is concerned with the dilemma facing a politician choosing between reducing unemployment and improving/preserving the quality level of the environment. A simple dynamic model for a state with a backward looking short-lived memory is presented.

Keywords: Environment; Unemployment; Dynamic Optimization; Political Business Cycle

1. Introduction

The paper studies a dynamic optimal choice in a political system. In particular, a politician's choice for reducing unemployment and improving and/or preserving the quality level of the environment is discussed when the politician or his/her political party in power wants to win at the next election in an environmentally conscious state with a backward looking short-lived memory.

We often observe that the public tends to believe that there is a trade-off between reducing unemployment and preserving the quality level of the environment. For example, see the February 1994 National Geographic magazine. This issue covered people in a small town in Idaho who believed that there was a trade-off relationship between reducing unemployment and preserving the quality level of environment and argued that the forest should be developed for the sake of reducing the high local unemployment level. And according to the Hotelling-Downs model, the politician is assumed to maximize the probability of reelection rather than the public interest [see Lane(1993), pp.81-82]. Therefore, regardless whether reducing unemployment and preserving the quality of the environment has truly a negative dynamic relation in an economy, the problem is that the politician's choice to gain popularity will be subject to public belief.

The problem of policy choices in the framework of the Hotelling-Downs model was first discussed by Nordhaus (1975). Nordhaus investigated a dynamic optimal decision between unemployment and inflation for an incumbent political party in order to win popularity.

The Nordhaus' model can be also applied to many economic policy choice problems. However, it is rare to see economic literatures considering a tradeoff relationship between reducing unemployment and improving/preserving the quality level of the environment, which we often find in actual policy-making practice. In this paper we consider a problem of policy choices between reducing unemployment and improving/preserving the quality level of the environment. In the next section, we apply Nordhaus' model after some generalizations for our problem of choice between unemployment and environment. The paper is concluded in Section 3.

2. A Dynamic Choice Model

A politician in an incumbent party will pursue policies that appeal to as many voters as possible so that the party can retain control of government at the next election.

Suppose a politician (or a political party) gain control of government or congress through election at time t=0, and the next election will be held at t=T period. Suppose that the politician choose only one fixed policy which will affect an electoral period from 0 to T. This policy will not spill over into the next electoral period. Voters are assumed to have a short-lived memory: *myopic and backward-looking*. Therefore, the later events from the beginning of the electoral period will be weighted more heavily over the given electoral period. Note that this weighting scheme is very different from the conventional forward-looking weights often used for economic decisions. Nordhaus (1975) employed this backward looking weight. While Nordhaus assumed in his model a linear vote function with respect to inflation, the voter's preference function is generalized in our model so that it is quadratic with respect to a proxy of unemployment level (y) and the shortfall of the quality level of environment (D) respectively.

People often believe that a tradeoff between reducing unemployment rate and preserving the quality level of the environment exists. A politician faced with re-election at the next electoral period maximizes the voter's preference function under the constraints. The expectation of the public the concerning the quality level of the environment are an important factor for the policy decision. We assume an expected-augmented version of tradeoff relation for the level of unemployment and the quality level of the environment and we adopt the adaptive expectation hypothesis about how expectations for the quality level of the environment are specifically formed.

Okun's law suggests that the level of unemployment (U) has a negative relationship with the current income level (Y) and therefore the unemployment level (U) can be proxied by the full employment income (Y_f) minus the current income level (Y). Let $y = Y_f - Y$.

Let us assume the following specific voting function and tradeoff relation between the shortfall of the quality level of the environment (D) and a proxy of unemployment level (y).

$$v(y, D) = \frac{1}{2}y^2 + \frac{1}{2}hD^2$$
$$D = (j - ky) + aD^e$$

Then the politician's problem will be

$$\min \int_0^T (\frac{1}{2}y^2 + \frac{1}{2}hD^2)e^{rt}dt \qquad (1)$$

(2)

subject to

$$D = (j - ky) + aD^e \qquad (3)$$
$$\dot{D}^e = b(D - D^e) \qquad (4)$$

where $y = Y_f - Y$, and $D = E - E_f$ is the shortfall of current environmental quality level (E) from the perfect environmental quality level (E_f). $\dot{D}^e$ is the expected level of shortfall in environmental quality and $\dot{D}^e \equiv \frac{dD^e}{dt}$. Note that $\dot{D}^e = b(D - D^e)$ is equivalent to $\dot{E}^e = b(E - E^e)$. $r > 0$ represents the rate of decaying memory for past events. The level of D is a function of the level of y and the expected level of D (D^e).

In the analysis, we assume the political party in power has the ability to implement any target rate of D at any time. We take D^e as a state variable, and D as a control variable since D affect y via equation (3). The value of y will be determined when the values of D and D^e are known and it can be viewed as a function of the other two variables.

The Lagrangian integrand function is

$$L = \frac{1}{2}(y^2 + hD^2)e^{rt} + \lambda_1(j - ky - D + aD^e) + \lambda_2(-bD + bD^e + \dot{D}^e) \quad (5)$$

where λ_1 and λ_2 are Lagrange multipliers. The Euler-Lagrange equations are

$$L_y - \frac{d}{dt}L_{y'} = ye^{rt} - \lambda_1 k = 0 \quad (6)$$

$$L_D - \frac{d}{dt}L_{D'} = hDe^{rt} - \lambda_1 - \lambda_2 b = 0 \quad (7)$$

$$L_{D^e} - \frac{d}{dt}L_{\dot{D}^e} = \lambda_1 a + \lambda_2 b - \frac{d}{dt}\lambda_2 = 0 \quad (8)$$

$$L_{\lambda_1} - \frac{d}{dt}L_{\lambda'_1} = j - ky - D + aD^e = 0 \quad (9)$$

$$L_{\lambda_2} - \frac{d}{dt}L_{\lambda'_2} = -bD + bD^e + \dot{D}^e = 0 \quad (10)$$

Solving the above equations simultaneously, we arrive at a more explicit form of the Euler equation. This signs of coefficients of the Euler equation is somewhat complicated. To resolve this difficulty, *let's set j = 0 and a = 1*. Then it reduces to

$$\ddot{D}^e + r\dot{D}^e + gD^e = 0. \tag{11}$$

where $g = \frac{(r-b)bhk^2}{hk^2+1}$ and r, b, k and $h > 0$. The general solution of this second order homogeneous differential equation will be

$$D^{e*}(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t} \tag{12}$$

where

$$r_1, r_2 = \frac{-r \pm \sqrt{r^2 - 4g}}{2}.$$

If $r_2 > 4g$, then the square root has always a positive numerical value and the characteristic roots are real and distinct. In this case, when $g < 0$, the characteristic roots are

$$r_1 > 0, r_2 < 0,$$

and when $g > 0$, the characteristic roots are

$$r_1 < 0, r_2 < 0.$$

Note $g < 0$ iff $r < b$, and $g > 0$ iff $r > b$.

If $r^2 < 4g$, the characteristic roots will be a pair of conjugate complex numbers.

$$r_1, r_2 = h \pm vi$$

where $h = \frac{-r}{2}$ *and* $v = \frac{\sqrt{4g - r^2}}{2}$. In this case of $r^2 < 4g$, the optimal path can be rewritten as

$$D^{e*}(t) = e^{ht}(A_3 cosvt + A_4 sinvt) \tag{13}$$

where $A_3 = A_1 + A_2, A_4 = A_1 - A_2$. Since h is always less than zero, the time path is convergent over time and it is characterized by damped fluctuation.

Assume the initial condition to be

$$D^e(0) = D_0^e = A_1 + A_2 > 0. \tag{14}$$

At the given T, two arbitrary constants can be definitized by using the following transversality condition:

$$F_{\dot{D}_e}|_{t=T} = 0,$$
$$\text{where } F = \frac{1}{2}(y^2 + hD^2)e^{rt}.$$

This transversality condition, after some simplification, can be written as

$$\dot{D}^e(T) + \zeta D^e(T) = 0. \tag{15}$$

where

$$\zeta = \frac{bhk^2}{hk^2 + 1}.$$

This equation (15) together with equation (12) yields

$$(r_1 + \zeta)A_1 e^{r_1 T} + (r_2 + \zeta)A_2 e^{r_2 T} = 0. \tag{16}$$

This transversality condition equation (16) simultaneously with the initial condition yields a following definite optimal path for D^e:

$$D^{e*}(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t} \tag{17}$$

where

$$A_1 = \frac{-D_0^e(r_2 + \zeta)e^{r_2 T}}{(r_1 + \zeta)e^{r_1 T} - (r_2 + \zeta)e^{r_2 T}}$$
$$A_2 = \frac{D_0^e(r_1 + \zeta)e^{r_1 T}}{(r_1 + \zeta)e^{r_1 T} - (r_2 + \zeta)e^{r_2 T}}.$$

Therefore, the optimal path for y from equations (3) and (4) will be

$$y^*(t) = -\frac{1}{kb}(r_1 A_1 e^{r_1 t} + r_2 A_2 e^{r_2 t}).$$

What will happen if t = T ? The optimal state of D^e at $t = T$ will be

$$D^{e*}(T) = 0. \qquad (18)$$

Therefore, the optimal environmental policy for a politician in the model is to preserve the quality level of the environment to the certain extent which the expected shortfall in the environmental quality reduces to zero at the terminal time T.

The optimal state of $y^*(T)$ will be:
if $r^2 > 4g$,

$$y^*(T) = \frac{-\sqrt{r^2 - 4g}}{kb[(r_1 + \zeta)e^{r_1 T} - (r_2 + \zeta)e^{r_2 T}]},$$

and if $r^2 < 4g$, then

$$y^*(T) = \frac{-\sqrt{4g - r^2 i}}{kb[(r_1 + \zeta)e^{r_1 T} - (r_2 + \zeta)e^{r_2 T}]}.$$

Note all terminal state variables, $D^{e*}(T)$ and $y^*(T)$ are endogenously determined. And the constants A_1 and A_2 depend on r_1, r_2 and ζ so that it make us difficult to identify the corresponding signs.

One of implications from the above analysis in which when terminal state variables are endogenously determined, the expected shortfall in the environmental quality has to reduce to zero at the terminal time. To resolve a problem in determining signs of constants in the above case, let us modify our model into the optimization problem with a fixed terminal state: $D_T^e = 0$. In this new model, a politician wishes to reduce the expected shortfall in the quality of the environment to zero.

Then the optimal path for D^e over time will be

$$D^{e*}(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t} \tag{19}$$

where

$$A_1 = \frac{-D_0^e e^{r_2 T}}{e^{r_1 T} - e^{r_2 T}}$$

$$A_2 = \frac{D_0^e e^{r_1 T}}{e^{r_1 T} - e^{r_2 T}}.$$

The coefficients, A_1 and A_2 in equation (19) are only different from those of equation (14). Besides this difference, optimal paths for D^e, y, and U are the same as the aforementioned fixed time case.

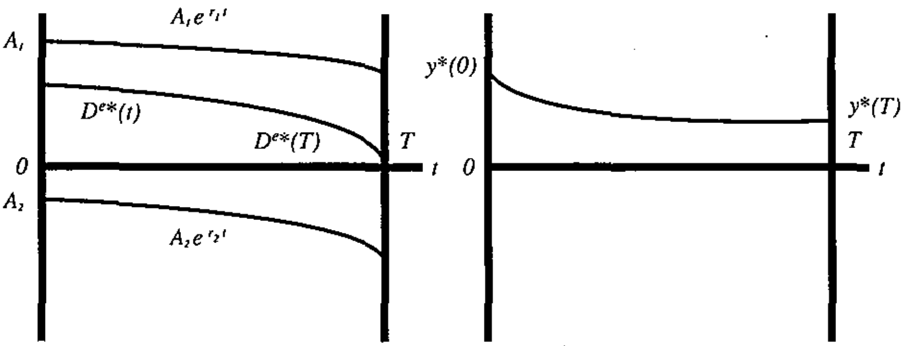
The signs of coefficients in this fixed terminal state problem case can be easily derived compared to the previous fixed time problem and are summarized as follows:

$r^2 > 4g$	$g > 0$	$r_1 < 0, r_2 < 0$	$A_1 < 0, A_2 > 0$
	$g < 0$	$r_1 > 0, r_2 < 0$	
$r^2 < 4g$	$g > 0$	$r_1, r_2 = \frac{-r \pm \sqrt{4g - r^2 i}}{2}$	$A_3 < 0, A_4 > 0$

Note A_1 always takes a positive number while A_2 is negative, regardless of values of r_1 and r_2.

The following figure shows the optimal paths for D^e and y.

Figure 1



The optimal path for D^e starts from $D_0 = A_1 + A_2 > 0$ and descends toward zero at t = T. Over the electoral period, the environmental quality level should be improved to such an extent that it equal to E_f. Note that this D^{e*} is a equidistant function of two exponential functions: $A_1 e^{r_1}$ and $A_2 e^{r_2}$. The optimal paths can be either concave or convex over time, depending on the two exponential component curves.

The optimal path for y is a simple inverse function of $\dot{D}^e_*$. Equating (3) and (4) gives

$$y^*(t) = -\frac{1}{kb}\dot{D}^{e*}$$

$$= -\frac{1}{kb}(r_1 A_1 e^{r_1 t} + r_2 A_2 e^{r_2 t}).$$

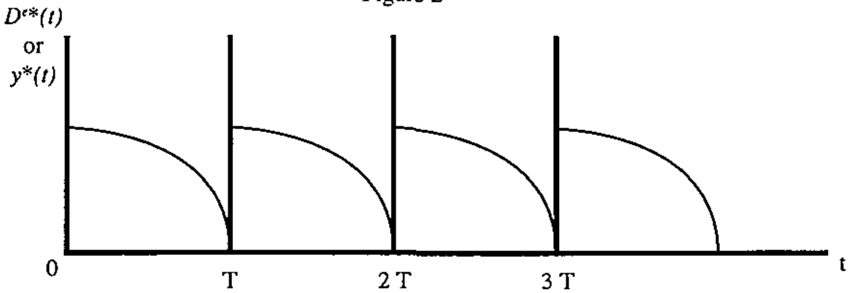
The proxy of unemployment level, y^* takes positive numbers at both initial and terminal periods. And $y^*(0)$ is greater than $y^*(T)$ when $r^2 > 4g$. Recall $y = Y_f - Y$. Therefore, the unemployment level has to decrease gradually over an electoral period. The rate of changes in the shortfall in the quality of the environment and the level of unemployment depends on the expectation adjustment coefficient (b), the rate of decaying memory for past events (r) and the relative weight attached to the shortfall in the quality of the environment (h).

3. Concluding Remarks

Our model shows that the optimal strategy to maximize popularity is: at the beginning of the electoral period, to raise the expected level of shortfall in the quality of the environment and the unemployment level as high as possible and then gradually decrease both over the entire electoral period.

This myopic optimal behavior generates a sawtoothed path for the expected level of the environmental quality and the level of unemployment across electoral periods.

Figure 2



The figure 2 indicates that a politician's myopic optimal behavior could create a vicious cycle in the quality level of the environment and the level of unemployment under an electoral system in our model.

Two most crucial assumptions in our model to reach this rather pessimistic view of a political system are that there are no policy spill-over into the next electoral period, and a society has a short-lived memory ($r > 0$).

This model implies that to stop the vicious cycle in the quality level of the environment and the level of unemployment the electoral system has to be modified so that voters can evaluate a politician/party across electoral periods and they have a long-lived memory.

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