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General Equilibrium under Factor Market Distortions [1/]

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1. Introduction

Factor market distortions (**FMD**) mean that the reward to a particular factor may not be equalized between the sectors even if the factors are allowed to move freely between them. These phenomena occur due to various reasons such as taxation, subsidies, imperfection in the market (labor unions, etc.) . This topic has been extensively discussed since Harberler's 1950 analysis. In the late 1960's and in the 70's, this topic enjoyed an "avalanche" (adopting a colorful word used by Magee, 1976) of researches especially in terms of the two-sector, two-factor framework. Magee (1976, p. xi) then remarked,

> " For years 2 × 2 trade theory had operated in what I felt was a theoretical vacuum: all of the standard theorems included the assumption of 'no imperfections in factor markets.' "

On the other hand, the basic apparatus necessary for studying the problem of FMD was already at hand through the work by Takayama (1963, 1964) and Jones (1965) that clarified the algebraic structure of the 2 × 2 general equilibrium model. Jones (1971)'s study on FMD using his 1965 framework was very important. In addition, Bhagwati, Srinivasan, Herberg, Kemp, Mayer, Magee, and Neary are some other economists who made important contributions on this topic.

By the mid-1970's, it had become clear that distortions in factor markets produce a number of paradoxical results. In surveying the results in this context of trade theory, Batra (1973, p.279) writes.

> "A Pandora's box of paradoxes is opened the moment the assumption of undistorted factor markets is relaxed."

FMD discussed in the literature are concerned either with a closed economy or with a small economy. As examples of the former, we have Harberger's (1962) study on corporate taxation, Lewis (1954) on the imperfect economy of labor migration, and Johnson-Mieszkowski (1970) on unionization.

With regard to the paradoxes associated with FMD for a **small open economy**, as Neary (1978a, p.671) summarized, there are "three principal paradoxes."

(i) Price-Output Response. An increase in the relative price of X vis à vis Y can lead to a fall in the output of X and a rise in the output of Y.

(ii) Lack of Correspondence between the Rybczynski and Stolper-Samuelson Theorems. The Rybczynski theorem holds when the factor intensity is defined in the (usual) physical sense, whereas the Stolper-Samuelson theorem holds when the factor intensity is defined in the "value sense."

(iii) Distortion-Output Response. An increase in a subsidy paid to one sector can reduce the output and the employment of that sector.

In contrast to the above paradoxes for a small open economy, we can point out at least the following five paradoxical comparative statics results for **a closed economy** under factor market distortions.

(i) Taste-Output Response. A change in tastes in favor of one commodity can lead to a decrease in the output of that commodity.

(ii) Taste-Price Response. A change in tastes in favor of one commodity can lead to an increase in the relative price of that commodity when the supply price curve is downward-sloping.

(iii) Taste-Factor Reward Response. A change in tastes in favor of the capital-intensive commodity can lead to an increase in the real wage rate and a fall in the rate of return on capital.

(iv) Distortion Output Response. An increase in the differential paid to either factor in one sector can lead to an increase in the output of that sector, i. e., an increase in the rate of subsidy paid to one sector can lead to a fall in the output of that sector.

(v) Distortion-Price Response. An increase in the differential paid to either factor in one sector can lead to a fall in the price of that sector: that is, an increase in the rate of subsidy paid to one sector can lead to an increase in the price of that sector.

These paradoxes for a closed economy are not well-known in the literature. However, the three paradoxes for a small open economy are only special cases of paradoxes for a closed economy in which demand is infinitely elastic (or the price elasticity of "demand price" is zero), as we shall show it in detail below.

The purpose of this paper is to obtain the proper perspective of the studies on FMD during the last 30 years. In particular, we clarify the circumstance in which these paradoxes appear, and such a circumstance can be removed under a plausible economic condition. In particular, we show that all these paradoxes disappear if and only if the LRE is Marshallian stable, and that the LRE is Marshallian stable if and only if it is stable under the capital adjustment process. Namely, we have an **equivalence theorem** with regard to (a) Marshallian stability, (b) the stability under the capital adjustment process, and (c) vanishing paradoxes. As a corollary to this result, we may conclude that the paradoxes all disappear if and only if the LRE is stable under the capital adjustment process. This implies that these paradoxes are only theoretical curiosa, as they will almost never be observed in real world economies. Neary's (1978a, b) well-known result that the paradoxes for a small open economy are necessarily associated with the instability of the capital adjustment process is only a part of the above results. Not only have we extended his analysis to a closed economy, but also we have clarified the proper perspective of his theorem, i.e., it is a part of the fundamental equivalence theorem, highlighting the importance of Marshallian stability. As shown elsewhere (Ide-Takayama, 1991, 1993a ,b), the scope of this theorem can be extended to economies with variable returns to scale.

In this context we may recall the celebrated distinction between short-run and long-run equilibria by Marshall (1920, esp. pp. 373-379). In the long-run equilibrium (**LRE**) à la Marshall, a long enough time is allowed so that capital is adjusted to its optimal scale. Along with this distinction, Marshall (1920, esp. pp. 345-347, pp. 805-806) developed what is known today as Marshallian stability, which is to be contrasted with Walrasian stability. As is well-known, the two stability conditions, Walrasian and Marshallian, have been distinguished in the literature, where the former is concerned with the price adjustment process, and the latter is concerned with the output adjustment process. The natural question is which of the two stability conditions we should choose for a particular application. No satisfactory answer has been given to this question in the literature. Although the stability conditions of these two processes are the same if the supply curve is upward-sloping, this is no longer the case if this curve is downward-sloping at the equilibrium point. In fact, the supply curve containing negatively-sloped portion(s) turns out to be at the heart of many problems including FMD and variable returns to scale.

Our equivalence theorem then answers the question of which stability condition we should choose: i.e., the equivalence between (a) and (b) provides a micro foundation to choose the Marshallian condition for the economies that involve production. As mentioned earlier, the same equivalence theorem holds for an economy under variable returns to scale. Namely, this theorem is not confined to the economy under FMD, but rather it can provide a unifying principle that prevails in economic theory encompassing different situations.

As is known by now, the description of an equilibrium being Marshallian stable but Walrasian unstable (or vice versa) is in fact meaningless, since the two stability criteria deal with the questions of two different dimensions, i.e., the Walrasian condition deals with pure exchange economy and the Marshallian condition is concerned with production (cf. Yasui, 1940; Newman, 1965; Takayama, 1985). Namely, the description of an equilibrium being Marshallian stable but Walrasian stable, etc. contains a "serious substantive error of muddling up exchanges with production." [2/]

A brief outline of this paper is in order now. Section 2 presents the basic model that follows the λ - θ approach introduced by Jones (1965). Section 3 and 4, in the context of a small open economy, show that the three principal comparative statics paradoxes will disappear if and only if the LRE is Marshallian stable, and that the LRE is Marshallian stable if and only if it is stable under the capital adjustment process. Section 5 finds the major comparative statics paradoxes for a closed economy. Section 6 shows that all such paradoxes will disappear if and only if the LRE is Marshallian unstable. Section 7 then shows that the LRE for a closed economy is Marshallian stable if and only if it is stable under the capital adjustment process. Section 8 is concerned with an application of the Marshallian condition to the pattern of specialization in a small open economy under FMD. In this section, we shall also analyze the effect of a change in distortion parameters. We shall show, for example, that subsidies to agriculture can lead to the disappearance of that sector. This paper is complete with four appendices. Appendices A establishes the shape of the supply price curve for the CES class of production functions. Appendix B, C, and D are technical notes, where B and D are concerned with the stability of capital allocation processes, and C deals with distortion-output responses.

2. Model

We consider an economy consisting of two industries (X and Y), where production functions are specified by,

$$X = F(L_X, K_X) = L_X f(k_x),\ k_x \equiv L_X / L_X.$$

$$Y = G(L_Y, K_Y) = L_Y g(k_y),\ k_y \equiv K_Y / L_Y.$$

where F and G are assumed to be homogeneous of degree one, and where L_j and K_j, respectively, signify the labor and the capital input in the j-th industry.

Let a_{ij} denote the quantity of factor i required to produce one unit of commodity j. The requirement that both factors are fully employed is given by,

$$a_{LX}X + a_{LY}Y = L, \quad a_{KY}X + a_{KY}Y = K, \tag{1}$$

where L and K, respectively, denote the endowment of labor and capital.

Let w_j and r_j, respectively, denote the (real) wage rate and the (real) rate of return on capital, measured in terms of commodity Y, which prevail in the j-th industry (j = x, Y). Imperfect mobility of factors, taxation, subsidies, unionization of labor, and other forms of distortions prevent the factor rewards from being equal between the two sectors. Let p be the price of commodity X in terms of Y. Then we have the following zero profit condition.

$$a_{LX}w_x + a_{KX}r_x = p, \quad a_{LY}w_y + a_{KY}r_y = 1. \tag{2}$$

These four equations in (1) and (2), in rate of change terms are shown below.

$$\lambda_{LX}\hat{X} + \lambda_{LY}\hat{Y} = \hat{L} - [\lambda_{LX}\hat{a}_{LX} + \lambda_{LY}\hat{a}_{LY}], \tag{1'-a}$$

$$\lambda_{KX}\hat{X} + \lambda_{KY}\hat{Y} = \hat{K} - [\lambda_{KX}\hat{a}_{KX} + \lambda_{KY}\hat{a}_{KY}], \tag{1'-b}$$

$$\theta_{LX}\hat{w}_x + \theta_{KX}\hat{r}_x = \hat{p} - [\theta_{LX}\hat{a}_{Lx} + \theta_{KX}\hat{a}_{KX}], \tag{2'-a}$$

$$\theta_{LX}\hat{w}_y + \theta_{KY}\hat{r}_y = 0 - [\theta_{LY}\hat{a}_{LY} + \theta_{KY}\hat{a}_{KY}], \tag{2'-b}$$

where (^) signifies the rate of change. λ_{ij} is the fraction of the i-th factor used in the j -th production, and θ_{ij} signifies the i-th cost share of the j-th industry.

It is assumed that LRE under FMD can be characterized by <u>constant</u> proportional differentials between the factor rewards in the two sectors.

$$w_y = \alpha w_x, \quad r_y = \beta r_x. \tag{3}$$

where the term "long-run" is to be understood in the Marshallian sense. In rate of change terms, (3) can be written as,

$$\hat{w}_y = \hat{w}_x + \hat{\alpha}, \quad \hat{r}_y = \hat{r}_x + \hat{\beta}. \tag{3'}$$

The coefficients of production, a_{ij}'s, are chosen so as to minimize the cost for each firm in the usual fashion. Further, defining the Allen elasticities of factor substitution of the j-th industry by σ_j, we may obtain:

$\hat{a}_{LX} = -\theta_{KX}\sigma_x\hat{\omega}_x$, $\hat{a}_{KX} = -\theta_{LX}\sigma_x\hat{\omega}_x$, $\hat{a}_{LY} = -\theta_{KY}\sigma_y\hat{\omega}_y$, $\hat{a}_{KY} = \theta_{LY}\sigma_y\hat{\omega}_y$, (4)

where ω_j be the wage-rental ratio ($\omega_j = w_j / r_j$) in the j-th industry.

Assume that distortion parameters are constant ($\hat{\alpha} = \hat{\beta} = 0$). Then we have, $\hat{w}_x = \hat{w}_y \equiv \hat{w}$, $\hat{r}_x = \hat{r}_y \equiv \hat{r}$, and $\hat{\omega}_x = \hat{\omega}_y \equiv \hat{\omega}$. Using (4) with (1') and (2') yields:

$$\begin{bmatrix} \lambda_{LX} & \lambda_{LY} \\ \lambda_{KX} & \lambda_{KY} \end{bmatrix} \begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix} = \begin{bmatrix} \hat{L} \\ \hat{K} \end{bmatrix} + \begin{bmatrix} \delta_L \\ -\delta_K \end{bmatrix} \hat{\omega}, \qquad (5)$$

$$\begin{bmatrix} \theta_{LX} & \theta_{KX} \\ \theta_{LY} & \theta_{KY} \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{r} \end{bmatrix} = \begin{bmatrix} \hat{p} \\ 0 \end{bmatrix}, \qquad (6)$$

where $\delta_L \equiv \lambda_{LX}\theta_{KX}\sigma_x + \lambda_{LY}\theta_{KY}\sigma_y$ and $\delta_K \equiv \lambda_{KX}\theta_{LX}\sigma_x + \lambda_{KY}\theta_{LY}\sigma_y$. The economic interpretations of δ_L and δ_K without FMD are provided by Jones (1965, p. 561). Let λ and θ be notations for the matrices of coefficients shown in (5) and (6). Since each row sum is unity, the determinants $|\lambda|$ and $|\theta|$ are given by,

$$|\lambda| = \lambda_{LX} - \lambda_{KX} = \lambda_{KY} - \lambda_{LY} = (k_y - k_x)L_X/K,$$

$$|\theta| = \theta_{KY} - \theta_{KX} = \theta_{LX} - \theta_{LY} = (\beta k_y - \alpha k_x)w_x r_x L_X L_Y/(pXY).$$

Hence we may assert:

$$|\lambda| \lesseqgtr 0 \quad \text{according to whether} \quad k_x \gtreqless k_y, \qquad (7\text{-a})$$

$$|\theta| \lesseqgtr 0 \quad \text{according to whether} \quad \alpha k_x \gtreqless \beta k_y. \qquad (7\text{-b})$$

Namely, the sign of $|\lambda|$ depends on the relative factor intensity in the <u>physical</u> sense, whereas the sign of $|\theta|$ depends on the relative factor intensity in the <u>value</u> sense. [3]

Let $A \equiv |\lambda||\theta|$. Then we have:

$A > 0$, if and only if $|\lambda|$ and $|\theta|$ are of the same sign. (8)

Namely $A > 0$ if and only if the physical and value factor intensity rankings coincide with each other.

Solving (5) for $\hat{X}$ and $\hat{Y}$, we obtain:

$$\hat{X} = [\mu_X\hat{\omega} + (\lambda_{KY}\hat{L} - \lambda_{LY}\hat{K})] / |\lambda|, \; \hat{Y} = [-\mu_Y\hat{\omega} + (\lambda_{LY}\hat{K} - \lambda_{KX}\hat{L})] / |\lambda|, \qquad (9)$$

where $\mu_X \equiv \lambda_{LY}\delta_K + \lambda_{KY}\delta_L > 0$ and $\mu_Y \equiv \lambda_{LX}\delta_K + \lambda_{KX}\delta_L > 0$. When $\hat{K} = \hat{L} = 0$, letting $z \equiv X / Y$, we obtain the following relations from (9),

$$\hat{X} = \mu_X\hat{\omega} / |\lambda|, \; \hat{Y} = -\mu_Y\hat{\omega} / |\lambda|, \; \hat{Z} = \mu\hat{\omega} / |\lambda|, \qquad (10)$$

where $\mu \equiv \mu_X + \mu_Y > 0$. Also solving (6) for $\hat{w}$ and $\hat{r}$, we obtain:

$$\hat{w} = \theta_{KY}\hat{p} / |\theta|, \hat{r} = -\theta_{LY}\hat{p} / |\theta|, \hat{\omega} = \hat{p} / |\theta|. \tag{11}$$

Combining the third equation of (11) with (10), we obtain:

$$\hat{X} = \mu_X\hat{p} / A, \hat{Y} = -\mu_Y\hat{p} / A, \tag{12-a}$$

$$\hat{Z} = \mu\hat{p} / A, \tag{12-b}$$

where $A \equiv |\lambda||\theta|$, and where we assume $A \neq 0$. If $A = 0$, we have $\hat{p} = 0$. In the absence of distortions ($\alpha = \beta = 1$), $A > 0$ always. If $A > 0$, X increases and Y decreases as p increases. With distortions, it is possible to have $A < 0$.

Define the **(long-run) supply price** as the price at which producers are just breaking even, and denote it as p_L. Namely, p_L is the minimum acceptable price for producers. In the general equilibrium context, it is the one which is obtained by the three conditions of full employment, efficient allocation of resources, and zero profit for each industry. Denote the (long-run) supply price relation by $p_L(Z)$. It is **long-run** in the sense that along this relation the wage rate and the rate of return on capital are equalized between the two sectors. Note that the p used above in fact refers to the long-run (LR) supply price, so that p in (12) can be replaced by p_L.

When we plot p_L over Z (measuring Z on the horizontal axis), the p_L-curve may not be upward-sloping in the usual way, as it depends on the sign of A. It is upward-sloping or downward-sloping depending on whether $A > 0$ or $A < 0$. Also, it can be upward-sloping in one region of Z and downward-sloping in another region. We define the **elasticity of the LR supply price** (ϵ) by,

$$\epsilon \equiv \hat{p} / \hat{Z} = A / \mu, \tag{12'}$$

or $A = \mu\epsilon$. Clearly, $\epsilon \gtreqless 0$ depending on whether $A \gtreqless 0$, and the LR supply price curve is upward- or downward-sloping depending on whether $\epsilon \gtreqless 0$.

3. Marshallian Stability and Comparative Statics for a Small Open Economy

To close the model, here we assume that the country is a **small open economy,** i.e., commodity price ratio (p^*) is a constant which is given exogenously by the rest of the world. We assume that p^*-line intersects the p_L-curve at least once. We now consider the following **Marshallian output adjustment process:** [4/]

$$\dot{Z} = a_1[p^* / p_L(Z) - 1] = \phi(Z),\ a_1 > 0, \tag{13}$$

where the dot signifies the time derivative. Namely, the output of X increases relative to that of Y if and only if the (world) demand price p^* exceeds the (domestic) supply price p_L. It is assumed that throughout the adjustment process, the endowment of each factor as well as distortion parameters (α and β) stay constant. Let Z^* be defined by $\phi(Z^*) = 0$, and assume that there exists a finite value Z^* which is positive. Z^* is a **long-run equilibrium** (LRE) à la Marshall. $Z^* > 0$ signifies incomplete specialization. Assuming away the knife-edge case of $\phi(Z^*) = 0$, Z^* is asymptotically (locally) stable <u>if and only if</u> $\phi'(Z^*) < 0$. From (13), we may readily compute,

$$\phi'(Z^*) = -a_1\epsilon / Z^*. \tag{14}$$

We call the LRE **Marshallian stable** if it is stable under (13). Ignoring the knife-edge case of $\phi'(Z^*) = 0$, we can obtain the following result from (14).

Proposition 1 The LRE is Marshallian stable <u>if and only if</u> $\epsilon > 0$.

Since $\epsilon > 0$ if and only if $A > 0$, we obtain the following by recalling (8).

Proposition 2 The LRE is Marshallian stable <u>if and only if</u> the <u>physical</u> and <u>value</u> factor intensity ranking coincide with each other.

Corollary: The price-output response is normal <u>if and only if</u> the physical and value factor intensity ranking coincide with each other.

We say that the price-output response is **normal** (resp. **perverse**) if a "small" increase in the relative price of one commodity leads to an increase (resp. fall) in the output of that commodity and a fall (resp. increase) in the other output. Since we have $\hat{X} / \hat{p} > 0$ and $\hat{Y} / \hat{p} < 0$ if and only if $A > 0$, we obtain the following result by using Prop. 1.

Proposition 3 The price-output response is normal or perverse, depending upon whether the LRE is Marshallian stable or unstable.

The shapes of the production possibility frontier (PPF) and the LR supply price curve have been discussed in the literature. In Appendix A, we shall obtain the latter in a transparent way. [6] In Fig. 1, we illustrate a possible LR supply price (p_L) curve. In the standard case in which there are no FMD, the PPF is strictly concave and the LR supply price curve is upward-sloping for all value of Z (in the usual way). However with FMD, the PPF can have strictly convex portion(s), and the p_L-curve can have downward-sloping portion(s), i.e., it can "oscillate" as in Fig. 1. From (12"), the p_L-curve is upward-sloping or downward-sloping depending on whether $A > 0$ or $A < 0$, or equivalently $\epsilon > 0$ or $\epsilon < 0$.

For a small open economy, p is fixed at p*. Thus, in Fig. 1, there are three LRE points, E_1, E_2, and E_3. Note that $Z > 0$ (so that $X > 0$ and $Y > 0$) at any of the three LRE, E_1, E_2 and E_3. We can easily show that E_1 and E_3 are Marshallian stable, and E_2 is Marshallian unstable. Hence from Fig. 1, we can

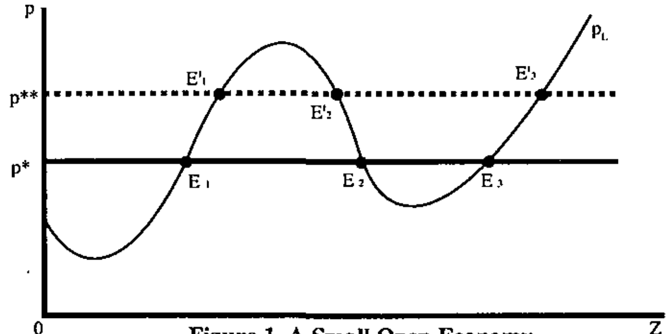


Figure 1 A Small Open Economy

see at once that the LRE is Marshallian stable or unstable depending on whether $\epsilon > 0$ or $\epsilon < 0$. Now suppose p* increases. Then for equilibria E_1 and E_3, Z increases, whereas for E_2, Z decreases. The former is the <u>normal</u> response, while the latter response is <u>perverse</u>. The latter means that a small increase in the relative price of X over Y (p*) <u>lowers</u> the output ratio X / Y. In summary, Props. 1 and 3 are illustrated in Fig. 1.

Assume incomplete specialization, and that this country exports Y and import X.

Assume also that she imposes a tariff on her imports of X, which raises the domestic p as $p = (1 + v)p^*$, where v is the ad valorem rate of tariff. Without loss of generality, we may suppose that X is more capital intensive than Y in the physical sense, i.e., $k_x > k_y$, or equivalently $|\lambda| < 0$. Assume the LRE is Marshallian stable so that $A > 0$ (Prop. 1), which in turn implies $|\theta| < 0$, given $|\lambda| < 0$. Then from (11), we may conclude: [7]

$$\hat{w} / \hat{p} < 0,\ \hat{r} / \hat{p} > 0,\ \hat{\omega} / \hat{p} < 0,\ \hat{w}^x / \hat{p} < 0,\ \hat{r}^x / \hat{p} > 0, \tag{15}$$

where w^x and r^x, respectively, signify the real wage rate and the real rate of return on capital (in terms of good X). Conversely, if (15) holds, then $|\theta| < 0$ so that $A > 0$ given $|\lambda| < 0$. (15) states that an increase in the relative price of the capital intensive commodity reduces the real reward to labor and increases the real reward to capital. As an alert reader would notice, this is nothing but the sign response obtained in the Stolper-Samuelson theorem. We say the Stolper-Samuelson sign pattern **normal** if it follows (15), and **perverse** if the signs in (15) are all reversed. Thus we obtain the following result.

Proposition 4 The Stolper-Samuelson sign pattern is normal or perverse, depending on whether the LRE is Marshallian stable or unstable.

In other words, if and only if the LRE is Marshallian stable, a (small) increase in the relative price of one commodity raises the real reward to the factor which is used in that industry more intensively (in the physical and value sense) and decreases the real reward to the other factor, when the endowment of each factor stays constant.

We now turn to Rybczynski theorem, the result which is dual to the Stolper-Samuelson theorem. Suppose that p is frozen at the world level p*, and consider a (small) increase in the endowment of one factor. Suppose that X is more capital intensive than Y in the physical sense ($k_x > k_y$), i.e., $|\lambda| < 0$ [cf. (7-a)]. From (11), $\hat{\omega} = 0$ when $\hat{p} = 0$. Hence from (9), we may obtain,

$$\hat{X} = (\lambda_{KY}\hat{L} - \lambda_{LY}\hat{K}) / |\lambda|,\ \ \hat{Y} = (\lambda_{LX}\hat{K} - \lambda_{KX}\hat{L}) / |\lambda|.$$

From this we have,

$$\hat{X} > 0, \hat{Y} < 0, \text{ if } \hat{K} > 0, \hat{L} = 0; \hat{X} < 0, \hat{Y} > 0, \text{ if } \hat{K} = 0, \hat{L} > 0, \quad (16)$$

when $|\lambda| < 0$. Namely, an increase in the capital endowment (K) keeping the labor endowment (L) constant increases the output of the capital intensive commodity and reduces the output of the labor intensive commodity. Similarly, an increase in L keeping K constant increases the output of the capital intensive commodity, and reduces the output of the labor intensive commodity. This is nothing but the familiar Rybczynski theorem. We say the Rybczynski sign pattern is **normal** if it follows (16), and **perverse** if the signs of $\hat{X}$ and $\hat{Y}$ in (16) are all reversed.

Now recall (11). Suppose that commodity X is more capital intensive than commodity Y in the value sense, i.e., $|\theta| < 0$ [cf. (7-b)]. Then from (11), we may obtain the Stolper-Samuelson sign pattern indicated. Thus, we at once obtain the following observation obtained by Jones (1971) and Magee (1976).

Observation: The Rybczynski theorem holds when the factor intensity relation is defined in the physical sense, while the Stolper- Samuelson theorem holds when the factor intensity relation is defined in the value sense.

This lack of correspondence between the Rybczynski and Stolper-Samuelson theorem is a principal paradox appearing in the context of FMD. [8] However, if and only if the equilibrium is Marshallian stable, A ($\equiv |\lambda||\theta|$) > 0 and the sign of $|\theta|$ is equal to that of $|\lambda|$, i.e., the factor intensity relation in the value sense coincides with that in the physical sense.

In summary, we obtain the following result.

Proposition 5

(i) The lack of correspondence between the Stolper-Samuelson and Rybczynski theorems disappears if and only if the LRE is Marshallian stable.

(ii) The Rybczynski sign pattern is normal or perverse, depending on whether the LRE is Marshallian stable or unstable.

The last principal paradox here is that of a perverse distortion-output response, which studies the effects of changes in α and β on outputs. When $\hat{\alpha} \neq 0$ and $\hat{\beta} \neq 0$, we have $\hat{w}_x \neq \hat{w}_y$, $\hat{r}_x \neq \hat{r}_y$, and $\hat{\omega}_x \neq \hat{\omega}_y$, in general. As will be shown in Appendix C, we may obtain:

$$\hat{Z} = (\delta_\alpha \hat{\alpha} + \delta_\beta \hat{\beta}) / (\epsilon\mu), \tag{17}$$

$$\delta_\alpha \equiv \theta_{LY}\mu_X\sigma_x + \theta_{LX}\mu_Y\sigma_y > 0,\ \delta_\beta \equiv \theta_{KY}\mu_X\sigma_x + \theta_{KX}\mu_Y\sigma_y > 0. \tag{18}$$

From this, we may at once conclude that <u>if and only if</u> $\epsilon > 0$,

$\hat{Z} / \hat{\alpha} > 0$ (when $\hat{\beta} = 0$) ; $\hat{Z} / \hat{\beta} > 0$ (when $\hat{\alpha} = 0$)

It can be shown easily that $\hat{Z} / \hat{\alpha} > 0$ if and only if $\hat{X} / \hat{\alpha} > 0$ and $\hat{Y} / \hat{\alpha} < 0$, and that $\hat{Z} / \hat{\beta} > 0$ if and only if $\hat{X} / \hat{\beta} > 0$ and $\hat{Y} / \hat{\beta} < 0$. Therefore, the distortion-output response relation can be summarized in Table 1.

	$\hat{\beta}$=0		$\hat{\alpha}$=0	
	$\hat{X}/\hat{\alpha}$	$\hat{Y}/\hat{\alpha}$	$\hat{X}/\hat{\beta}$	$\hat{Y}/\hat{\beta}$
e>0	+	-	+	-
e<0	-	+	-	+

Table 1 Distortion-Output Responses

We say the distortion-output response is **normal** (resp. **perverse**), if an increase in the differential paid on either factor in one sector lowers (resp. increases) the output of that sector; that is, if an increase in the rate of subsidy paid to one sector increases (resp. reduces) the output of that sector. Then from Table 1, we may conclude that the distortion-output response is normal or perverse depending on whether $\epsilon > 0$ or $\epsilon < 0$. Then recalling Prop. 1, we at once obtain the following result for a small open economy.

Proposition 6 The distortion-output response is normal or perverse, depending on whether the LRE is Marshallian stable or unstable.

4. Marshallian Stability and Intersectoral Capital Mobility for a Small Open Economy

In the Marshallian short-run equilibrium (SRE), capital is sector specific, and in the Marshallian LRE, the capital stock in each sector is adjusted to the optimal scale. Assuming that the endowment of each factor and the distortion parameters α and β stay constant and setting $\hat{p} = 0$ by the small country assumption, we can postulate the following capital adjustment process:

$$\dot{K}_X = b_1[\beta r_x / r_y - 1], \quad b_1 > 0 \tag{19}$$

where we have $r_y = \beta r_x$ in the LRE. Here, it is assumed that labor is adjusted and the SRE is achieved instantaneously, so that $w_y = \alpha w_x$ holds always.

The adjustment process (19) can be written in the form of

$$\dot{K}_X = b_1[\beta r_x(K_X) / r_y(K_X) - 1] \equiv \psi(K_X). \tag{19'}$$

As shown in Appendix B, we may compute $\psi'(K_X^*)$ as follows.

$$\psi_X'(K^*) = -\frac{b_1 \mu \varepsilon}{\lambda_{KY} B K_X^*}, \text{ where } B \equiv \lambda_{LX}\theta_{KY}\sigma_x + \lambda_{LY}\theta_{KX}\sigma_y > 0. \tag{20}$$

Here K_X^* signifies the LRE value of K_x; i.e., $\psi'(K_X^*) = 0$. Assuming away the knife-edge case of $\psi'(K_X^*) = 0$, K_X^* is stable if and only if $\psi'(K_X^*) < 0$. Hence the LRE under the adjustment process (19') is stable <u>if and only if</u> $\epsilon > 0$. Thus we may obtain the following result.

Proposition 7 The LRE for a small open economy is Marshallian stable, <u>if and only if</u> it is stable under the capital adjustment process.

This proposition means that the Marshallian output adjustment process may be interpreted as the shadow of the capital adjustment process projected onto the output space. It happily weds Marshall's two celebrated concepts, "Marshallian stability" and the "long-run equilibrium," both which appear in his <u>Principles</u>. [2/] By this proposition, we may conclude that Marshallian unstable equilibria are only theoretical curiosa as they can almost never be observed in real world economies. Thus, by way of Props. 3, 5, and 6, the three comparative statics paradoxes with regard to (a) the price-output response, (b) the lack of correspondence between the Rybczynski and Stolper-Samuelson theorems, and (c) distortion-output response, all become theoretical curiosa. We may now state the following result due to Neary (1978a, b).

Corollary: The three comparative statics paradoxes occur if and only if the LRE is unstable under the capital adjustment process.

Remark Although Neary (1978a, pp. 678-679) conjectured that there might be a close relationship between Marshallian stability and the "long-run supply curve," he has neither formulated nor obtained the result showing a close relationship between Marshallian stability and the stability of the long-run equilibrium under the capital adjustment process as stated in Proposition 7.

5. Normal and Perverse Results, and Marshallian Stability for a Closed Economy

In the above, we assumed that the country in question is a small open economy. A similar analysis can be applied to a closed economy. We shall now proceed our analysis to such an economy. Instead of assuming p = constant, we impose the following inverse demand function that is homothetic.

$$p = h(z, t), \;\; \partial h / \partial z < 0, \qquad (21)$$

where t denotes the shift parameter representing a change in tastes. In Marshall's terminology, p signifies the **demand price**, the maximum price that consumers are willing to pay. Consider a change in tastes in favor of good X relative to good Y, and assume $\partial h / \partial t > 0$. Differentiation of (21) yields,

$$\hat{p} = -\eta\hat{Z} + \tau\hat{t}, \qquad (22)$$

where $\eta \equiv -(\partial h / \partial z)(z / p)$ (**the elasticity of demand price**) and $\tau \equiv (\partial h / \partial t)(t / p)$, and where $\eta > 0$ and $\tau > 0$. Assume that at the LRE, $Z > 0$ (i.e., both goods are produced). Then combining (22) with (12-b) and recalling $A = \mu\epsilon$, we obtain,

$$\hat{Z} / \hat{t} = \tau / (\eta + \epsilon). \qquad (23)$$

We say that the taste-output response is **normal** (resp. **perverse**) if a change in tastes in favor of one commodity results in an increase (resp. fall) in the output of that commodity relative to the other commodity. Then from (23), we obtain the following result.

Proposition 8 The taste-output response is normal or perverse depending on whether $\eta + \epsilon > 0$ or $\eta + \epsilon < 0$.

Next combining (23) with (12-b), we may obtain,

$$\hat{p} / \hat{t} = \tau\epsilon / (\eta + \epsilon). \qquad (24)$$

We say that the taste-price response is **normal** if change in tastes in favor of one commodity results in a fall in the relative price of that commodity over the other commodity in the case that the supply price curve is downward-sloping. Similarly, the taste-price response is **perverse**, if a change in tastes favoring one commodity results in an increase in the relative price of that commodity in the case that the supply price curve is downward-sloping. Using (24), we then obtain the following result.

Proposition 9 The taste-price response is normal or perverse, depending on whether $\eta + \epsilon > 0$ or $\eta + \epsilon < 0$.

Combining (24) with (11) and recalling $\epsilon = A / \mu = |\lambda||\theta| / \mu$, we may obtain,

$$\hat{w} / \hat{t} = \theta_{KY}\tau|\lambda| / (\eta + \epsilon)\mu, \quad \hat{r} / \hat{t} = - \theta_{LY}\tau|\lambda| / (\eta + \epsilon)\mu, \qquad (25)$$

Table 9.2 summarizes the effect of a change in tastes upon factor prices that can be obtained from (25).

	$\eta+\varepsilon>0$		$\eta+\varepsilon<0$	
	$\hat{w}/\hat{t}$	$\hat{r}/\hat{t}$	$\hat{w}/\hat{t}$	$\hat{r}/\hat{t}$
$k_x>k_y$	-	+	+	-
$k_x<k_y$	+	-	-	+

Table 2 Taste Factor Price Responses

We say that the taste factor price response is **normal**, if a change in tastes in favor of the capital intensive (resp. labor intensive) commodity in the physical sense lowers (resp. increases) the real wage rate and increases (resp. lowers) the real rate of return on capital. On the other hand, we say the taste factor price response is **perverse** if the response opposite to the normal response is obtained. From Table 2, we then obtain the following result.

Proposition 10 The taste factor price response is normal or perverse, depending on whether $\eta + \epsilon > 0$ or $\eta + \epsilon < 0$.

Next, we investigate the effects of a change in distortion parameters (α and β) on output and prices, when tastes remain unchanged. In this case $\hat{\alpha} \neq 0$ and $\hat{\beta} \neq 0$, so that $\hat{w}_x \neq \hat{w}_y$, $\hat{r}_x \neq \hat{r}_y$, and $\hat{\omega}_x \neq \hat{\omega}_y$. As shown in Appendix C, the effects of a change in distortion parameters on Z and p can be obtained as (26) and (27), and such effects are summarized in Table 3.

$$\hat{Z} = (\delta_\alpha\hat{\alpha} + \delta_\beta\hat{\beta}) / (\eta + \epsilon)\mu, \tag{26}$$

$$\hat{p} = - \mu(\delta_\alpha\hat{\alpha} + \delta_\beta\hat{\beta}) / (\eta + \epsilon)\mu, \tag{27}$$

where $\delta\alpha$ and $\delta\beta$ are defined in (18).

	$\hat{\beta}>0$		$\hat{\alpha}<0$	
	$\hat{Z}/\hat{\alpha}$	$\hat{p}/\hat{\alpha}$	$\hat{Z}/\hat{\beta}$	$\hat{p}/\hat{\beta}$
$\eta+\varepsilon>0$	+	-	+	-
$\eta+\varepsilon<0$	-	+	-	+

Table 3 The Effects of a Change on Distortion Parameters

The distortion-output response is said to be **normal** (resp. **perverse**), if an increase in the differential paid on either factor in one sector lowers (resp. increases) the output of that sector; that is, if an increase in the rate of subsidy paid to one sector increases (resp. reduces) the output of the sector. Similarly, the distortion-price response is said to be **normal** (resp. **perverse**) if an increase in the differential paid on either factor in one sector raises (resp. reduces) the price of that sector, i.e., if an increase in the rate of subsidy paid to one sector lowers (resp. increases) the price of that sector. Then from Table 3, we obtain the following result.

Proposition 11

(i) The distortion-output response is normal or perverse, depending on whether $\eta + \epsilon > 0$ or $\eta + \epsilon < 0$.

(ii) The distortion-price response is normal or perverse, depending on whether $\eta + \epsilon > 0$ or $\eta + \epsilon < 0$.

6 . Marshallian Stability and Paradoxes for a Closed Economy

We now analyze the role of Marshallian stability. To this end, recall that the LR supply price curve relation is denoted by $p = p_L(Z)$, in which we may recall (12''). The p_L-curve is upward- or downward-sloping depending on whether $\epsilon > 0$ or $\epsilon < 0$. Assuming constant preferences, we may write demand curve as,

$$p = h(z, t) \equiv p_D(Z), \qquad (21')$$

$p_D(Z)$ signifies the demand price curve. With this, (22) can be rewritten as,

$$\hat{Z} = - \eta \hat{p}_D. \qquad (22')$$

We now postulate the Marshallian adjustment process by,[9]

$$\dot{Z} = a_2[p_D(Z) / p_L(Z) - 1] = \Phi(Z), \quad a_2 > 0. \qquad (28)$$

Namely, Z increases or decreases depending upon whether the demand price exceeds or falls short of the supply price. Assume that there exists $Z^* > 0$ such that $\Phi(Z^*) = 0$, at which $p_D = p_L = p^*$, where $Z^* > 0$ signifies an incomplete specialization LRE in the present context.

Fig. 2 illustrates the LRE points and their Marshallian stability property. The p_D-curve signifies the demand price curve. It is always downward-sloping. On the other hand, the shape of the supply price curve depends on the sign of ϵ. Panels a and b, respectively, illustrate the case in which $\epsilon > 0$ for all Z and $\epsilon < 0$ for all Z. Namely

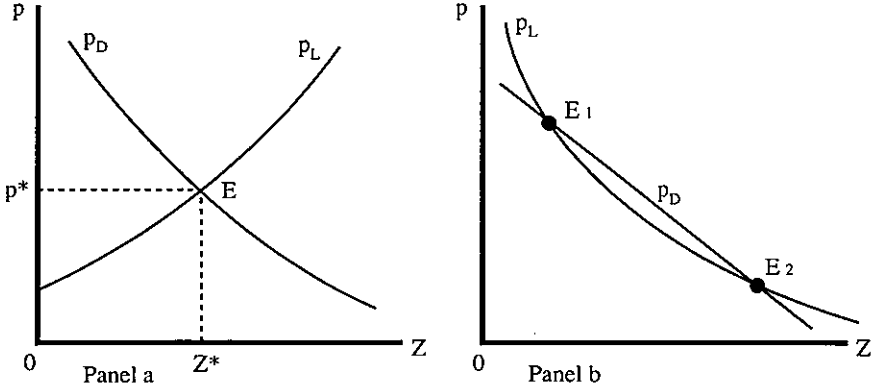


Figure 2 Illustration of Marshallian Stability

in Panel a, the supply price curve is uniformly upward-sloping, and in Panel b, it is uniformly downward-sloping. It is possible that $\epsilon > 0$ for a certain region of Z and $\epsilon < 0$ for another region of Z as was illustrated in Fig. 1. In Panel a, the LRE point E is unique and (globally) Marshallian stable. In Panel b, there are two LRE points E_1 and E_2, where E_2 is (locally) Marshallian stable and E_1 is Marshallian unstable.

We may obtain the Marshallian stability condition in mathematical terms. Assuming away the knife-edge case of $\Phi'(Z^*) = 0$, LRE Z^* is stable if and only if $\Phi'(Z^*) < 0$. Since we can obtain,

$$\Phi'(Z^*) = \frac{a_2}{p^*}\left[\frac{dp_D}{dZ} - \frac{dp_L}{dZ}\right],$$

we may conclude from (12") and (22') that

$$\Phi'(Z^*) < 0 \text{ if and only if } \eta + \epsilon > 0.$$

Thus we obtain the following result.

Proposition 12 The LRE is Marshallian stable <u>if and only if</u> $\eta + \epsilon > 0$.

Thus, if the supply price curve is upward-sloping ($\epsilon > 0$), Z* is always stable. When it is negatively-sloped ($\epsilon < 0$), Z* is stable if the demand price curve is steeper than the demand price curve ($\eta < |\epsilon|$). Combining Prop. 2 with Props. 8-11, we obtain the following conclusion.

Proposition 13 The taste-output, taste-price, taste factor price, distortion-output, and distortion-price responses are all normal <u>if and only if</u> the LRE is Marshallian stable, i.e., $\eta + \epsilon > 0$.

Remark: If there are no factor market distortions ($\alpha \equiv \beta \equiv 1$), the p_L-curve is always upward-sloping, the equilibrium is always Marshallian stable, the taste-output, taste factor-price, and taste-price responses are all normal.

Fig. 3 illustrates the effects of a change in tastes in favor of X relative to Y upon outputs and prices. The demand price curve after the change in tastes is

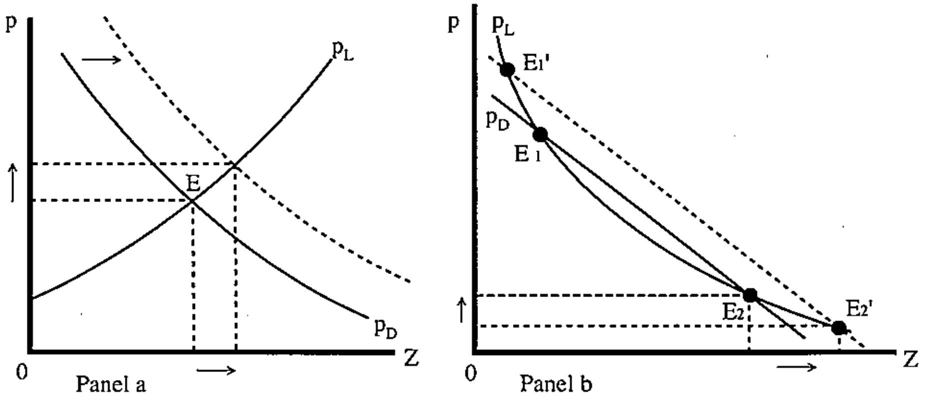


Figure 3 An Illustration of a Change in Tastes in Favor of Commodity X

illustrated by the dashed curves, where we consider only the cases in which $\epsilon > 0$ for all Z (Panel a) and $\epsilon < 0$ for all Z (Panel b). In Panel a, the LRE (E) is (globally) Marshallian stable, and both the taste-output and taste-price responses are normal. Namely, a change in tastes in favor of X increases Z and p. In Panel b, there are two LRE points, E_1 and E_2, where E_1 is Marshallian unstable and E_2 is Marshallian stable. At point E_2 in Panel b, the taste-output response is normal in which case we have $\eta + \epsilon > 0$, while at E_1, it is perverse in which case we have $\eta + \epsilon < 0$.

7. Marshallian Stability and Intersectoral Capital Mobility for a Closed Economy

Here, as in the small open economy case, we show that the stability of adjustment process from the SRE to the LRE is closely related to Marshallian stability for a closed economy. Assume that tastes and distortion parameters remain constant. As before, we then postulate the following adjustment process.

$$\dot{K}_X = b_2[\beta r_x / r_y - 1], b_2 > 0, \qquad (19)$$

where we have $r_y = \beta r_x$ in the LRE.

Using the model described earlier, we may express the RHS of (19) as a function of K_X alone. We then define function ψ by,

$$\psi(K_X) \equiv b_2[\beta r_x(K_X) / r_y(K_X) - 1]. \qquad (29)$$

Assuming away the case of $\Psi'(K_X^*) = 0$, the LRE is stable if and only if $\Psi'(K_X^*) < 0$. As shown in Appendix D, the expression for $\psi'_X(K^*)$ can be obtained as,

$$\Psi'(K_X^*) = -b_2(\eta + \varepsilon)\mu / (\lambda_{KY}\theta_y B_5 K_X^*), \qquad (30)$$

where $B_5 > 0$ is defined in Appendix D, and it is reduced to B when $\eta = 0$.

From (30), we may then conclude that the LRE is stable, if and only if $\eta + \epsilon > 0$. Then recalling Prop. 12, we may obtain the following remarkable result for a closed economy under FMD.

Proposition 14 The LRE (Z*) is Marshallian stable (i.e., $\eta + \epsilon > 0$), if and only if it is the stable under the capital adjustment process.

Again, Prop. 14 means that the output adjustment process may be interpreted as the shadow of the capital adjustment process projected onto the output space. By this proposition, the paradoxes for a closed economy discussed earlier are only theoretical curiosa, since they are almost never observed in real world economies. In other words, we have the following corollary.

Corollary: The perverse taste-output, taste-price, taste factor price, distortion-output, and distortion-price responses are all obtained only for the LRE that is unstable under the capital adjustment process.

Remark: The case of a small open economy can be regarded as a special case of the present analysis in which $\eta = 0$. In this case, condition $\eta + \epsilon > 0$ is reduced to $\epsilon > 0$. Thus, Props. 1 and 7 can, respectively, be considered special cases of Props. 12 and 14 which $\eta = 0$. Also, the above corollary includes Neary's theorem as a special cases in which $\eta = 0$.

8. Pattern of Specialization and a Change in Factor Market Distortion Parameters for a small Open Economy

So far we have concentrated our attention on an incomplete specialization equilibrium for both the cases of a small open economy and a closed economy. However, the above discussion also sheds light on the pattern of specialization. We shall analyze this problem by using the case of a small open economy. In Figs. 4 and 5, we consider three possible shapes of the p-curve. [11]

In Panel a of Fig. 4, point E is a unique Marshallian stable LRE in which

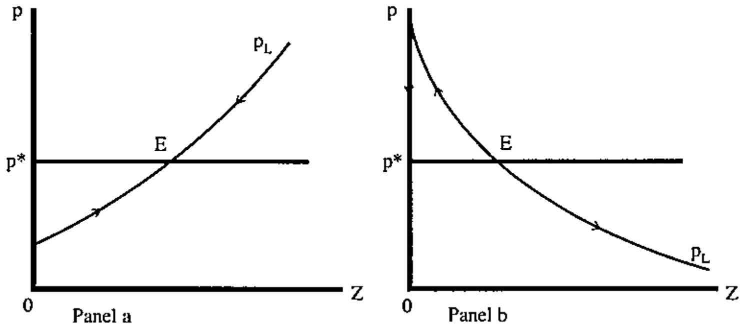


Figure 4 Pattern of Specialization I

the country produces both commodities under trade, regardless of whether $p^A > p^*$ or $p^A < p^*$, where $p^A < p^* \equiv (p_x / p_y)^A$ denote the LRE price ratio under autarky. In Panel b of Fig. 4, E is a Marshallian unstable LRE. In this case, the country specializes in the production of Y if $p^A > p^*$, and that she specializes in the production of X if $p^A > p^*$. Namely, the country completely specializes in the production of a commodity in which she has comparative advantage. [11] The LRE in Panel b of Fig. 4 will almost never be observed (except for the knife-edge case of $p^A = p^*$). The comparative statics results with respect to such an E have little meaning. On the other hand, comparative statics results are meaningful with respect to point E in Panel a.

In both Panels a and b of Fig. 5, E_1 is the only stable incomplete specialization LRE in which both commodities are produced under trade. If $p^A > p^*$ as in Panel a, the autarkic equilibrium point should be either A_1 or A_2. In this case, the country will always be led to incomplete specialization equilibrium

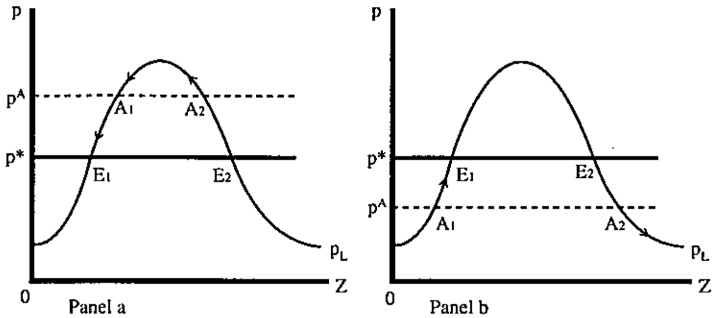


Figure 5 Pattern of Specialization II

(E_1) under free trade. If $p^A < p^*$, as in Panel b, the country will be led to an incomplete specialization equilibrium (E_1) under free trade if the autarkic equilibrium point is given by A_1, while she completely specializes in the production of X if the autarkic equilibrium point is given by A_2.

The above discussion provides an interesting application on the effect of a change in distortion parameter(s). Suppose that $\alpha = \beta = 1$ initially. The LR supply price curve is upward-sloping as illustrated in Panel a of Fig. 6, in which case the country produces both commodities. Now suppose that α / β [12/] change (via subsidies to agricultural capital, taxation of the industrial wage, unionization of industrial workers, etc.). After such change(s), the shape of the supply price curve will change. In general, the supply price curve can have many troughs and peaks as was indicated in Fig. 1. If the production functions are of the CES type, we can show that the shape of the supply price curve is any one of the shapes in Panels a, b, and c of Fig. 6) can occur. [13] Suppose that the supply price curve becomes uniformly downward-sloping. Then the country is led into complete specialization, and one industry (which could be a subsidized agriculture, a unionized industry, etc.) will disappear. This case is illustrated by Panel b of Fig. 6. If the supply price curve is bell-shaped after the change in α / β (cf. Panel c), the country is led either to incomplete specialization

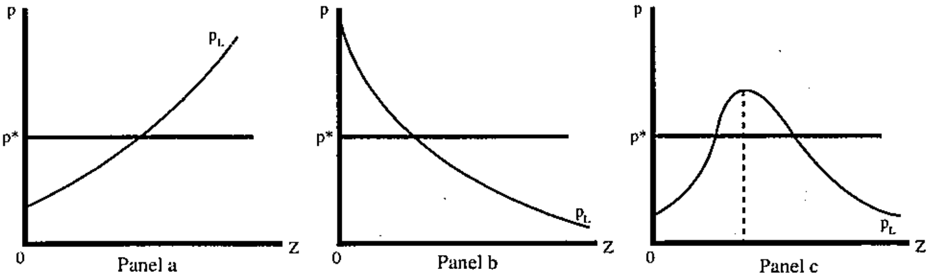


Figure 6 The Implication of a Change in (a/b) to the Pattern of Specialization

or to complete specialization, depending on the magnitude of α / β after the change. For the latter case, one industry would again disappear. In the above we assumed that $\alpha = \beta = 1$ initially to avoid clutter. Note that the above argument does not depend on this assumption. [14]

Appendix A : The shape of the Long-Run Supply Price Curve and the Production Possibility Frontier under Distortions

The Cobb-Douglas case provides a useful introduction to the discussion in this section. We specify the production functions by,

$$X = L_X^{\xi} K_X^{1-\xi},\ Y = L_Y^{\zeta} K_Y^{1-\zeta},\ 0 < \xi < 1,\ 0 < \zeta < 1. \qquad \text{(A-1)}$$

In this case, we have,

$$\theta_{LX} = \xi,\ \theta_{KX} = 1 - \xi,\ \theta_{LY} = \zeta,\ \theta_{KY} = 1 - \zeta,\ |\theta| = \theta_{LX} - \theta_{LY} = \xi - \zeta.$$

Also, via the usual cost minimization, we have,

$$k_x = (1 - \xi)\omega_x / \xi,\ k_y = (1 - \eta)\omega_y / \zeta = \gamma(1 - \zeta)\omega_x / \zeta,$$

where $\gamma \equiv \alpha / \beta$. Thus we obtain,

$$k_x - k_y = C\omega_x / (\xi\zeta), \text{ where } C \equiv (1 - \xi)\zeta - \gamma(1 - \zeta)\xi. \qquad \text{(A-2)}$$

Note that $|\lambda| \lesseqgtr 0$ depending upon whether $C \gtreqless 0$. Without loss of generality, we may assume that $|\theta| < 0$ ($\zeta > \xi$): in other words, the X-industry is relatively more capital-intensive than the Y-industry in the value sense.

If there are no FMD ($\alpha = \beta = \gamma = 1$), $C > 0$ and $|\lambda| < 0$, so that $A \equiv |\lambda||\theta| > 0$. Thus the p_L-curve is always upward-sloping. However with distortions, $\gamma \neq 1$, in general, and hence the sign of C is indeterminate. If $C > 0$, $A > 0$ uniformly, while if $C < 0$, $A < 0$ uniformly. In particular if $\gamma \leqq 1$, then $C = 0$ so that $A > 0$ uniformly. Thus we may obtain the following result.

Proposition A.1 In the Cobb-Douglas case, the p_L-curve is either uniformly upward-sloping or uniformly downward sloping. In particular, if the X-industry is more capital intensive than the Y-industry in the value sense <u>and</u> if $\alpha \leqq \beta$, then the p_L-curve is uniformly upward-sloping.

Next, we consider the CES case. In this case, we have:

$$k_x = c_1\omega_x^{\sigma_x},\ k_y = c_2\omega_y^{\sigma_y} = c_2(\gamma\omega_x)^{\sigma_y}, \qquad \text{(A-3a)}$$

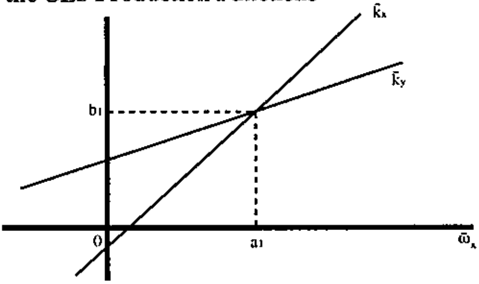
$$\theta_{LX} = [1 + c_1\omega_x^{(\sigma_x - 1)}]^{-1},\ \theta_{LY} = [1 + c_2(\gamma\omega_x)^{(\sigma_y - 1)}]^{-1}. \qquad \text{(A-3b)}$$

Taking the logarithm of the equations in (A-3a) and putting bars (-) over the log values (i.e., $\bar{k}_x \equiv \log k_x$, etc.), we have,

$$\bar{k}_x = \bar{c}_1 + \sigma_x\bar{\omega}_x,\quad \bar{k}_y = \bar{c}_2 + \sigma_y(\bar{\gamma} + \bar{\omega}_x), \qquad \text{(A-4)}$$

We focused our attention on the case of $\sigma_x \neq \sigma_y$. (The case of $\sigma_x = \sigma_y$ can be analyzed analogously, in which case the conclusion is similar to the Cobb-Douglas case). Without a loss of generality, we may assume $\sigma_x > \sigma_y$. We may illustrate (A-4) in Fig. 7,[15] where at a_1 the factor intensity reversal occurs. We denote the value of $\bar{k}_x = \bar{k}_y$ when $\bar{\omega}_x = a_1$ as b_1. Clearly, the values of a_1 and b_1 can be positive, zero, or

Figure 7 The Factor Intensity Relation for the CES Production Functions



negative.

Since the factor intensity ranking in the physical sense differs from that in the values sense under factor-market distortions ($\gamma \neq 1$), we have, $|\theta| \neq 0$ and yet $|\lambda| = 0$. The value of a_1 (at which $|\lambda| = 0$) is obtained from (A-4) by setting $\bar{k}_x = \bar{k}_y$ as,

$$a_1 \equiv (\bar{c}_2 - \bar{c}_1 + \sigma_y \bar{\gamma}) / (\sigma_x - \sigma_y). \qquad \text{(A-5)}$$

Recall $|\theta| = \theta_{LX} - \theta_{LY}$. Substituting (A-3b) into $|\theta| = 0$ we obtain,

$$\bar{\omega}_x = a_2, \text{ where } a_2 \equiv [\bar{c}_2 - \bar{c}_1 - (1 - \sigma_y)\bar{\gamma}] / (\sigma_x - \sigma_y). \qquad \text{(A-6)}$$

Namely, $|\lambda| = 0$ when $\bar{\omega}_x = a_1$ and $|\theta| = 0$ when $\bar{\omega}_x = a_2$. Combining (A-5) with (A-6), we obtain the following remarkable relation.

$$a_1 - a_2 = \bar{\gamma} / (\sigma_x - \sigma_y), \text{ where } \bar{\gamma} \equiv \log(\alpha / \beta). \qquad \text{(A-7)}$$

Since $\sigma_x > \sigma_y$ by assumption, we may conclude from (A-7),

$$a_1 \gtreqless a_2 \text{ depending upon whether } \alpha \gtreqless \beta. \qquad \text{(A-8)}$$

We first focus our attention on <u>the case of $\alpha > \beta$</u>, in which case we have, $a_1 > a_2$. Clearly, $|\lambda| > 0$ for $\bar{\omega}_x < a_1$ and $|\lambda| < 0$ for $\bar{\omega}_x > a_1$. Also recalling,

$$|\theta| = w_x r_x L_X L_Y (k_y - \gamma k_x) / (pXY\beta),$$

we may conclude that $|\theta| > 0$ for $\bar{\omega}_x < a_2$ and $|\theta| < 0$ for $\bar{\omega}_x > a_2$. We may then summarize the signs of $|\lambda|$, $|\theta|$, and $A \equiv |\lambda||\theta|$ in Table 4.

	$\bar{\omega}_X < a_2$	$a_2 < \bar{\omega}_X < a_1$	$a_1 < \omega_X$
$\lvert\lambda\rvert$	+	+	-
$\lvert\theta\rvert$	+	-	-
A	+	-	+

Table 4 The Signs of $|\lambda|$, $|\theta|$, and A when $\alpha > \beta$

Define b_1, b_2, and b_3 for the present case of $\alpha > \beta$ by,

$b_1 \equiv \bar{k}_x$ $(=\bar{k}_y)$ when $\bar{\omega}_x = a_1$, $b_2 \equiv \bar{k}_y$ when $\bar{\omega}_x = a_2$, $b_3 \equiv \bar{k}_x$ when $\bar{\omega}_x = a_2$.

We may illustrate the values of b_1, b_2, and b_3 in Fig. 8, since they play an important role in our analysis. Though we assume $a_1 > 0$ and $b_1 > 0$ in Fig. 9-8 (which need not be the case), this does not affect our subsequent conclusions.

Let k be the capital-labor endowment ratio ($k \equiv K/L$) and let $\bar{k} \equiv \log k$. Depending on the size of $\bar{k}$, there are four cases that we need to consider. In these four cases, the sign of A can be determined by using Table 4, and referring to Fig. 8, we may obtain the possible shapes of the p_L-curve as follows.

(Case i): $b_1 < \bar{k}$

In this case we have, $|\lambda| < 0$, $|\theta| < 0$ and $A > 0$ always. Hence by (12'), the p_L-curve is uniformly upward-sloping (cf. Panel a of Fig. 9).

(Case ii): $b_2 < \bar{k} < b_1$

In this case we have, $|\lambda| > 0$, $|\theta| < 0$ and $A < 0$, always. Hence by (15), the p_L-curves is uniformly downward-sloping (cf. Panel b of Fig. 9).

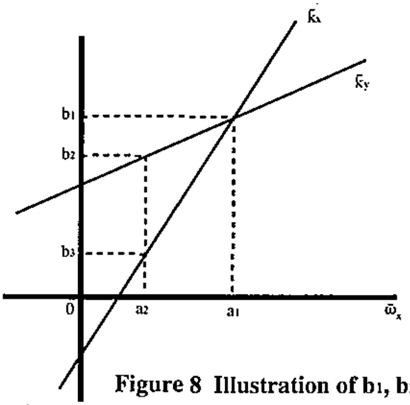


Figure 8 Illustration of b1, b2, and b3

(Case iii): $b_3 < \bar{k} < b_2$

In the case $|\lambda| > 0$ always, while $|\theta| > 0$ if $\bar{\omega}_x < a_2$ and $|\theta| < 0$ if $a_2 < \bar{\omega}_x < a_1$. Hence $A > 0$ for $\bar{\omega}_x < a_2$, and $A < 0$ for $a_2 < \bar{\omega}_x < a_1$. Then we may conclude that the p_L-curve is bell-shaped (cf. Panel c of Fig. 9), since $\hat{Z} = \delta\hat{\omega} / |\lambda|$ by (11) and (12-b).

(Case iv): $\bar{k} < b_3$

In this case we have $|\lambda| > 0$, $|\theta| > 0$ and $A > 0$, always. Hence, by (12') the p_L-curve is uniformly upward-sloping (cf. Panel a of Fig. 9).

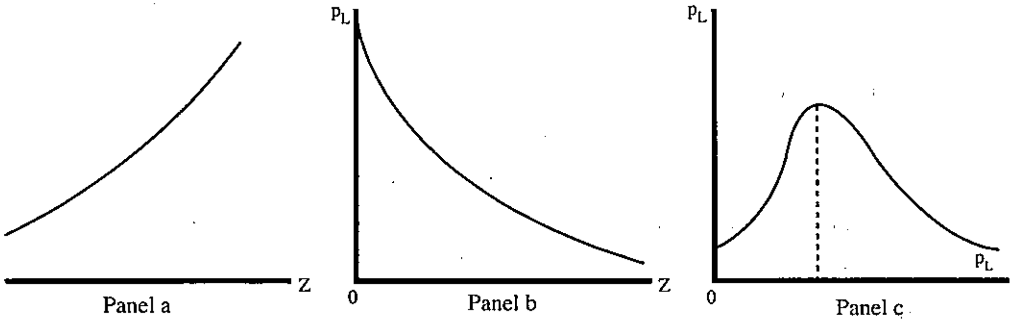


Figure 9 The Three Shapes of the Supply Price Curve under Distortions

We now turn to <u>the case of $\alpha < \beta$</u>, in which case we have $a_1 > a_2$ by (A-8). In this case, the signs of $|\lambda|$, $|\theta|$, and A can be determined as in Table 5.

	$\bar{\omega}_X<a_1$	$a_1<\bar{\omega}_X<a_2$	$a_2<\omega_X$
$\lvert\lambda\rvert$	+	-	-
$\lvert\theta\rvert$	+	+	-
A	+	-	+

Table 5 The Signs of $|\lambda|$, $|\theta|$ and A when $\alpha < \beta$

Define b_1', b_2', and b_3' for the present case of $\alpha < \beta$ as,

$b_1' \equiv \bar{k}_x \ (= \bar{k}_y)$ when $\bar{\omega}_x = a_1$, $b_2' \equiv \bar{k}_y$ when $\bar{\omega}_x = a_2$, $b_3' \equiv \bar{k}_x$ when $\bar{\omega}_x = a_2$.

The values of b_1', b_2', and b_3' can be illustrated in Fig. 10.

Again, depending on this size of $\bar{k}$, there are four cases that need to be considered. In these four cases, the signs of A can be determined as follows by using Table 5 and referring to Fig. 10. The shapes of the p_L-curve can then be determined

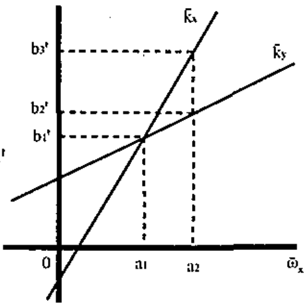


Figure 10 Illustration of b1', b2', and b3'

accordingly.

(Case i) : $\hat{k} > b_3'$

In this case we have, $|\lambda| < 0$, $|\theta| < 0$ and $A > 0$ always. Hence by (12'), the p_L-curve is uniformly upward-sloping (cf. Panel a of Fig. 9),

(Case ii) : $b_2' < \hat{k} < b_3'$

In this case we have, $|\lambda| < 0$, $|\theta| > 0$ if $a_1 \bar{\omega}_x < a_2$ and $|\theta| < 0$ if $a_2 < \bar{\omega}_x$. Thus $A < 0$ for $a_1 < \bar{\omega}_x < a_2$ and $A > 0$ for $a_2 < \bar{\omega}_x$. Then we may conclude that the p_L-curve is bell-shaped (cf. Panel c of Fig. 9), since $\hat{Z} = \delta\hat{\omega} / |\lambda|$ by (11) and (12-b).

(Case iii) : $b_1' < \hat{k} < b_2'$

In this case we have $|\lambda| < 0$, $|\theta| > 0$ and $A < 0$. Hence, by (12') the p_L-curve uniformly downward-sloping (cf. Panel b of Fig. 9).

(Case iv) : $\hat{k} < b_1'$

In this case we have $|\lambda| > 0$, $|\theta| > 0$, and $A > 0$. Hence, by (12') the p_L-curve is uniformly upward-sloping (cf. Panel a of Fig. 9).

In summary, we may obtain the following result.

Proposition A.2 If the production function of each industry is of the CES type with $\sigma_x > \sigma_y$, then the p_L-curve has either of the following three shapes: a) uniformly upward-sloping, b) uniformly downward-sloping, or c) bell-shaped, depending on the size of the capital-labor endowment ratio.

Remark: If $\sigma_x < \sigma_y$ instead, then the third possibility (the bell-shaped p_L-curve) is reversed: i.e., the p_L-curve will be U-shaped. This should be obvious as p measures the price of X in terms of Y.

The shapes of the PPF and the p_L-curve under the restriction of FMD have been discussed in the literature by Johnson (1966), Herberg-Kemp (1971), Herberg-Kemp-Magee (1971), and others. Our study here is to obtain the shape of the p_L-curve in a much simpler way than Herberg-Kemp-Magee (1971), and others.

Appendix B: The Stability of the Capital Allocation Process for a Small Open Economy[16]

Since $w_y = \alpha w_x$ and $r_y \neq \beta r_x$ in the short-run, we have $\hat{w}_y = \hat{w}_x (\equiv \hat{w})$, $\hat{r}_y \neq \hat{r}_x$ with $\hat{\alpha} = \hat{\beta} = 0$, which in turn implies $\hat{\omega}_y \neq \hat{\omega}_x$. Substituting (4) in the text into (2') and setting $\hat{p} = 0$, we may obtain,

$$\theta_{LX}\hat{w} + \theta_{KX}\hat{r}_x = 0, \qquad \text{(B-1a)}$$

$$\theta_{LY}\hat{w} + \theta_{KY}\hat{r}_y = 0. \qquad \text{(B-1b)}$$

Recalling that $a_{KX} \equiv K_X / X$, $a_{LX} \equiv L_X / X$, and $a_{KY} \equiv K_Y / Y$, we obtain the following relations from (4) in the text.

$$\hat{K}_X - \hat{L}_X = \sigma_x(\hat{w} - \hat{r}_x), \qquad \text{(B-2a)}$$

$$\hat{K}_Y - \hat{L}_Y = \sigma_y(\hat{w} - \hat{r}_y). \qquad \text{(B-2b)}$$

Note that the full-employment condition (1) in the text can be rewritten as,

$$\lambda_{LX} + \lambda_{LY} = 1, \lambda_{KX} + \lambda_{KY} = 1$$

From this we may obtain,

$$\lambda_{LX}\hat{L}_X + \lambda_{LY}\hat{L}_Y = 0, \lambda_{KX}\hat{K}_X + \lambda_{KY}\hat{K}_Y = 0$$

when $\hat{L} = \hat{K} = 0$. This can equivalently be written as,

$$\hat{L}_Y = - (\lambda_{LX} / \lambda_{LY})\,\hat{L}_X,\ \hat{K}_Y = - (\lambda_{KX} / \lambda_{KY})\,\hat{K}_X. \qquad \text{(B-3)}$$

Eliminating w from (B-1a) and (B-2a), and from (B-1b) and (B-2b), we obtain,

$$\hat{K}_X - \hat{L}_X = - (\sigma_x / \theta_{LX})\,\hat{r}_x, \qquad \text{(B-4a)}$$

$$\hat{K}_Y - \hat{L}_Y = - (\sigma_y / \theta_{LY})\,\hat{r}_y. \qquad \text{(B-4b)}$$

Substituting (B-3) into (B-4b), we obtain,

$$(\lambda_{LX} / \lambda_{LY})\, \hat{L}_X - (\lambda_{KX} / \lambda_{KY})\, \hat{K}_X = -(\sigma_y / \theta_{LY})\, \hat{r}_y. \qquad \text{(B-4'b)}$$

Eliminating $\hat{L}_X$ from (B-4a) and (B-4'a), we obtain,

$$|\lambda|\hat{K}_X = -\lambda_{LY}\lambda_{KY}\, [(\lambda_{LX}\sigma_x / \lambda_{LY}\theta_{LX})\, \hat{r}_x + (\sigma_y / \theta_{LY})\, \hat{r}_y], \qquad \text{(B-5)}$$

where we may recall $|\lambda| \equiv \lambda_{LX}\lambda_{KY} - \lambda_{KX}\lambda_{LY}$. Also from (B-1), we have,

$$(\theta_{LY}\theta_{KX})\, \hat{r}_x = (\theta_{KX}\theta_{KY})\, \hat{r}_y. \qquad \text{(B-6)}$$

From (B-5) and (B-6), we may obtain,

$$\hat{K}_X = -\frac{\lambda_{KY}B}{\theta_{LX}\theta_{KY}|\lambda|}\hat{r}_x,\ \hat{K}_X = -\frac{\lambda_{KY}B}{\theta_{LY}\theta_{KY}|\lambda|}\hat{r}_y, \qquad \text{(B-7)}$$

where $B \equiv \lambda_{LX}\theta_{KY}\sigma_x + \lambda_{LY}\theta_{KX}\sigma_y > 0$. From (B-7), we at once obtain,

$$\hat{r}_x / \hat{K}_X = -\theta_{LX}\theta_{KY}\, |\lambda| / (\lambda_{KY}B),\ \ \hat{r}_y / \hat{K}_X = -\theta_{LY}\theta_{KX}\, |\lambda| / (\lambda_{KY}B). \qquad \text{(B-8)}$$

From (19'), we may obtain the following equation.

$$\psi'(K_X^*) = b_1[\hat{r}_x / \hat{K}_X - \hat{r}_y / \hat{K}_X] / K_X^*.$$

Substituting (B-8) into this, and recalling $|\theta| \equiv \theta_{LX}\theta_{KY} - \theta_{LY}\theta_{KX}$ and $A \equiv |\lambda||\theta| = \mu\epsilon$, we obtain (20) in the text.

Appendix C: Distortion-Output Responses for a Closed Economy and for a Small Open Economy

We substitute (4) into (1') and (2'), set $\hat{K} = \hat{L} = 0$, and use (3') to obtain,

$$\lambda_{LX}\hat{X} + \lambda_{LY}\hat{Y} = \delta_L\hat{\omega}_x + \lambda_{LY}\theta_{KY}\sigma_y\, (\hat{\alpha} - \hat{\beta}), \qquad \text{(C-1a)}$$

$$\lambda_{KX}\hat{X} + \lambda_{KY}\hat{Y} = -\delta_K\hat{\omega}_x + \lambda_{KY}\theta_{LY}\sigma_y\, (\hat{\alpha} - \hat{\beta}), \qquad \text{(C-1b)}$$

$$\theta_{LX}\hat{w}_x + \theta_{KX}\hat{r}_x = p, \qquad \text{(C-2a)}$$

$$\theta_{LY}\hat{w}_x + \theta_{KY}\hat{r}_x = -(\theta_{LY}\hat{\alpha} + \theta_{KY}\hat{\beta}). \qquad \text{(C-2b)}$$

Solving (C-1) for $\hat{X}$ and $\hat{Y}$, we obtain,

$$\hat{Z}\ (= \hat{X} - \hat{Y}) = [\mu\hat{\omega}_x + (\lambda_{LY}\theta_{KY} + \lambda_{KY}\theta_{LY})\, \sigma_y\, (\hat{\alpha} - \hat{\beta})] / |\lambda|. \qquad \text{(C-3)}$$

Also solving (C-2) for $\hat{w}_x$ and $\hat{w}_y$, we may obtain,

$$\hat{w}_x = [\,\theta_{KY}\hat{p} + \theta_{KX}(\theta_{LY}\hat{\alpha} + \theta_{KY}\hat{\beta})] / |\theta|, \qquad \text{(C-4a)}$$

$$\hat{r}_x = -[\theta_{LY}\hat{p} + \theta_{LX}(\theta_{LY}\hat{\alpha} + \theta_{KY}\hat{\beta})] / |\theta|, \tag{C-4b}$$

$$\hat{\omega}_x = [\hat{p} + (\theta_{LY}\hat{\alpha} + \theta_{KY}\hat{\beta})] / |\theta|; \tag{C-4c}$$

Substituting (C-4c) into (C-3) and using (22) in the text with $\hat{t} = 0$ (tastes remaining unchanged), we may obtain,

$$\hat{Z} = (\delta_\alpha\hat{\alpha} + \delta_\beta\hat{\beta}) / (\eta + \epsilon)\mu, \tag{C-5}$$

where δ_α and δ_β are the same as the ones defined in (18) in the text. Substituting (C-5) into (C-3) and using (22) in the text with $\hat{t} = 0$, we also obtain,

$$\hat{p} = -\eta(\delta_\alpha\hat{\alpha} + \delta_\beta\hat{\beta}) / (\eta + \epsilon)\mu, \tag{C-6}$$

(C-5) and (C-6), respectively, correspond to (26) and (27) in the text.

The small open economy case is obtained by setting $\eta = 0$. Setting η in (B-5), we obtain,

$$\hat{Z} = (\delta_\alpha\hat{\alpha} + \delta_\beta\hat{\beta}) / (\epsilon\mu), \tag{C-7}$$

which corresponds to (17) in the text.

Appendix D: The stability of the Capital Allocation Process for a Closed Economy

As in Appendix B, we have, $\hat{w}_y = \hat{w}_x$, $\hat{r}_y \neq \hat{r}_x$. Using (4) and (2'), we obtain,

$$\theta_{LX}\hat{w} + \theta_{KX}\hat{r}_x = \hat{p}, \tag{D-1a}$$

$$\theta_{LY}\hat{w} + \theta_{KY}\hat{r}_y = 0, \tag{D-1b}$$

where $\hat{w} \equiv \hat{w}_x = \hat{w}_y$. Note that (B-2) holds as it is. Solving (D-1a) and (B-2a) for $\hat{w}$ and $\hat{r}_x$, we may obtain,

$$\hat{w} = (\theta_{KX}/\sigma_x)(\hat{K}_X - \hat{L}_X) + \hat{p}, \quad \hat{r}_x = -(\theta_{LX}/\sigma_x)(\hat{K}_X - \hat{L}_X) + \hat{p}. \tag{D-2}$$

Also solving (D-1b) and (B-2b) for $\hat{w}$ and $\hat{r}_y$, we have,

$$\hat{w} = (\theta_{KY}/\sigma_y)(\hat{K}_Y - \hat{L}_Y), \quad \hat{r}_y = -(\theta_{LY}/\sigma_y)(\hat{K}_Y - \hat{L}_Y). \tag{D-3}$$

Note that (B-3) holds as it is. Using (B-3), we may rewrite (D-3) as,

$$\hat{w} = (\theta_{KY}/\sigma_y)[(\lambda_{LX}/\lambda_{LY})\hat{L}_X - (\lambda_{KX}/\lambda_{KY})\hat{K}_X], \tag{D-3'a}$$

$$\hat{r}_y = (\theta_{LY}/\sigma_y)[(\lambda_{KX}/\lambda_{KY})\hat{K}_X - (\lambda_{LX}/\lambda_{LY})\hat{L}_X]. \tag{D-3'b}$$

Next differentiating the production functions of X and Y, we may obtain,

$$\hat{X} = \theta_{LY}\hat{L}_X + \theta_{KX}\hat{K}_X, \qquad \text{(D-4a)}$$

$$\hat{Y} = -\theta_{LY}(\lambda_{LX}/\lambda_{LY})\hat{L}_X - \theta_{KY}(\lambda_{KX}/\lambda_{KY})\hat{K}_X, \qquad \text{(D-4b)}$$

where we use (B-3) to obtain (D-4b). Using (D-4) and (22) in the text with $\hat{t} = 0$, we obtain,

$$-\hat{p} = \eta[(\theta_{LX} + \theta_{LY}(\lambda_{LX}/\lambda_{LY})]\hat{L}_X + \eta\,[\theta_{KX} + \theta_{KY}(\lambda_{KX}/\lambda_{KY})]\,\hat{K}_X. \qquad \text{(D-5)}$$

Substituting (D-5) into (D-2), we get:

$$\hat{w} = B_1\hat{K}_X - B_2\hat{L}_X, \qquad \text{(D-6a)}$$

$$\hat{r}_x = -B_3\hat{K}_X + B_4\hat{L}_X, \qquad \text{(D-6b)}$$

where

$$B_1 \equiv \theta_{KX}/\sigma_x - \eta\{\theta_{KX} + \theta_{KY}(\lambda_{KX}/\lambda_{KY})\},$$

$$B_2 \equiv \theta_{KX}/\sigma_x + \eta\{\theta_{LX} + \theta_{LY}(\lambda_{LX}/\lambda_{LY})\},$$

$$B_3 \equiv \theta_{LX}/\sigma_x + \eta\{\theta_{KX} + \theta_{KY}(\lambda_{KX}/\lambda_{KY})\},$$

$$B_4 \equiv \theta_{LX}/\sigma_x - \eta\{\theta_{LX} + \theta_{LY}(\lambda_{LX}/\lambda_{LY})\}.$$

Equating (D-3'a) with (D-6a), we obtain:

$$\hat{L}_X = \frac{\lambda_{LY}B_6}{\lambda_{KY}B_5}\hat{K}_X, \qquad \text{(D-7)}$$

where

$$B_5 \equiv B + \eta(\lambda_{LY}\theta_{LX} + \lambda_{LX}\theta_{LY})\,\sigma_x\sigma_y > 0, \quad B \equiv \lambda_{LX}\theta_{KY}\sigma_x + \lambda_{LX}\theta_{LY}\sigma_y > 0,$$

$$B_6 \equiv C - \eta(\lambda_{KY}\theta_{KX} + \lambda_{KX}\theta_{KY})\,\sigma_x\sigma_y, \quad C \equiv \lambda_{KX}\theta_{KY}\sigma_x + \lambda_{KY}\theta_{KX}\sigma_y > 0,$$

Here B is the same as the one used in Appendix B [cf. (B-7)]. Substituting (D-7) into (D-6b) and (D-3'b), we obtain,

$$\hat{r}_x = (\lambda_{LY}B_4B_6 - \lambda_{KY}B_3B_5)\,\hat{K}_X/(\lambda_{KY}B_5), \qquad \text{(D-8a)}$$

$$\hat{r}_y = \theta_{LY}(\lambda_{KX}B_5 - \lambda_{LX}B_6)\,\hat{K}_X/(\lambda_{KY}B_5\sigma_y). \qquad \text{(D-8b)}$$

From (29) in the text, we obtain,

$$\psi'(K_X^*) = b_2\,[\hat{r}_x/\hat{K}_X - \hat{r}_y/\hat{K}_X]\,/\,K_X^*.$$

Substituting (D-8) into this, we obtain (30) after some tedious manipulations.

Footnotes

1. This paper is developed from Ide-Takayama (1988a, b, c; 1990).
2. See Newman (1965, p. 107). Both Marshall (1920) and Walras (1926) had theories of production as well as that of pure exchange, and they both recognized that the price adjustment process is appropriate for the theory of exchange and that the outoput adjustment process is appropriate for the theory of production. Walras emphasized the theory of exchange and Marshall emphasized the theory of production in which he coined the celebrated concepts of short-run and long-run equilibria. Walras used graphical devices in the theory of exchange, but not in the theory of production. On the other hand, Marshall employed such devices in the theory of production, but not in the theory of pure exchange.
3. If $k_x > k_y$, the X-industry is relatively more capital intensive than the Y-industry in the **physical sense**, and if $\alpha k_x > \beta k_y$, then the X-industry is relatively more capital intensive than the Y-industry in the **value sense**. Similarly, X is relatively more labor intensive in the physical sense and in the value sense if $k_x < k_y$ and $\alpha k_x < \beta k_y$, respectively. See Johns (1971) and Magee (1976, p. 22).
4. This process corresponds to Aoki's (1970) exposition of the Marshallian process, which he describes as follows. "Suppose that an industry with excess of supply price over demand price expands and an industry with excess of demand price over supply price contracts,... (p.100). On the other hand, his development of the Marshallian theory in the latter part of his paper (e.g., p.106) is different from ours.
5. Thus Neary (1978a, pp. 673-674) states,"This shows that a reallocation of capital into sector X will reduce the proportional gap between the rentals in two sectors, if and only if the product of the two determinants, $|\lambda|$ and $|\theta|$, is positive; in other words, if and only if the rankings of the two sectors by physical and value factor intensities are the same."
6. It can be shown that if the production functions of both sectors are of the Cobb-Douglas type, the LR supply price curve is either uniformly upward-sloping or uniformly downward-sloping. It can also be shown that if the production functions of both sectors are of the CES type, then a third possibility can occur. Since it would be immaterial to call either industry X (or Y), we may assume $\sigma_x > \sigma_y$ without a loss of generality. Then the third possibility is a bell-shaped LR supply price curve. Namely, there exists a $z_t > 0$ such that the

LR supply price curve is upward-sloping for $z < z_I$ and downward-sloping for $z > z_I$. In general, the p_L-curve can oscillate as indicated in Fig. 9-1.

7. To obtain the signs for $\hat{w}^x$ and $\hat{w}^y$, note $\hat{w}^x = \hat{w} - \hat{p}$ and $\hat{r}^x = \hat{r} - \hat{p}$, and we obtain,

   $\hat{w}^x = \theta_{KX}\hat{p} / |\theta|$, $\hat{r}^x = -\theta_{LX}\hat{p} / |\theta|$, so that $\hat{w}^x / \hat{p} < 0$, $\hat{r}^x / \hat{p} > 0$, given $|\theta| < 0$.

8. Neary (1978a, pp. 671-672) explains this lack of correspondence as follows.

   "Each of these theorems continues to hold in isolation, but the former [the Rybczynski theorem] must be expressed in terms of physical factor intensities, and the latter [the Stolper-Samuelson theorem] in terms of value factor intensities. Hence, if labor force growth at constant commodity prices increases the output of good X, an increase in the relative price of X assuming a constant labor force will reduce rather than decrease the real wage: More surprisingly still, a country may be capital abundant relative to the rest of the world, and exporting its physically capital-intensive commodity ... and yet a protection-induced increase in the domestic price of the import-competing good will reduce the real return of the scarce factor (labor)." (the emphasis is in the original).

9. For Marshall's discussion of "Marshallian stability," see his Principles (1920, especially, pp. 345-347 and pp. 805-806). For his discussion of long-run equilibrium, see Principles (1920, especially, pp. 373-379). For the clarification of the confusion between the Marshallian stability and the Walrasian stability, see Newman (1965, esp., pp. 106-108), Takayama (1985), and Ide-Takayama (1993b).

10. For the shape of the p_L-curve, see Appendix A. Recall also footnote 6.

11. In both diagrams in Fig. 4, $Z = 0$ means $Z = 0$, or the country completely specializes in the production of Y, while when $Z \to \infty$, $Y \to 0$, so that the country specializes in the production of X. In Panel a, the p_L-curve becomes vertical as 0A at $Z = 0$. If $p^* < 0A$, then the country completely specializes in the production of Y. In Panel b, the p_L-curve becomes vertical at $Z = 0$, which is that portion of the vertical axis above point A.

12. Since only the factor price ratio matters in all the discussion in this chapter, a change in α and/or β can be summarized by a change in α / β.

13. See Fig .9 in Appendix A.

14. In real world economies, the production functions can be more "flexible" than the ones dictated by CES functions. There thus can be many troughs and peaks in the supply price

curve, so that the tendency towards complete specialization can be stopped. However, we can still argue that subsidies to agriculture can cause its decay, though not to the point of its extinction.

15. From this diagram, it should be clear that if the elasticities of factor substitution are constant, then the factor intensity reversal never occurs or occurs only once, depending on whether $\sigma_x = \sigma_y$ or $\sigma_x \neq \sigma_y$. This conclusion is obtained by using a similar diagrammatical technique introduced in Takayama (1963, pp. 80-82; 1972, pp. 82-85.).

16. While the present discussion is obtained as a special case of the closed economy discussion in Appendix C by setting $\eta = 0$, it would still be useful as the discussion in Appendix C is quite tedious.

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